

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2026
(First Semester)

Branch- STATISTICS

PROBABILITY THEORY AND RANDOM VARIABLE

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	If $P(A)=0.3$, $P(B)=0.5$, and $P(A \cap B)=0.1$, then $P(A \cup B)=$ (a) 0.6 (b) 0.7 (c) 0.8 (d) 0.9	K1	CO1
	2	Bayes' theorem is used to find (a) Prior probability (b) Posterior probability (c) Dependent probability (d) Sample probability	K2	CO1
2	3	The total probability of all possible values of a discrete random variable is (a) 0 (b) 1 (c) ∞ (d) Undefined	K1	CO2
	4	If X and Y are independent, then $E[XY] =$ (a) $E[X]+E[Y]$ (b) $E[X] \times E[Y]$ (c) $E[X-Y]$ (d) None of these	K2	CO2
3	5	If X and Y are independent, then $f(x,y)=$ (a) $f_X(x) + f_Y(y)$ (b) $f_X(x) \times f_Y(y)$ (c) $f_X(x) / f_Y(y)$ (d) None of these	K1	CO3
	6	The cumulative distribution function of (X,Y) is (a) $P(X \leq x, Y \leq y)$ (b) $P(X = x, Y = y)$ (c) $P(X \geq x, Y \geq y)$ (d) None of these	K2	CO3
4	7	The probability generating function (PGF) is applicable to (a) Continuous random variables (b) Discrete random variables (c) Both types (d) None	K1	CO4
	8	The characteristic function $\phi_X(t)$ of X is always (a) Real (b) Imaginary (c) Complex (d) Zero	K2	CO4
5	9	The Law of Large Numbers is related to (a) Averages (b) Variance (c) Probability density (d) Skewness	K1	CO5
	10	The Central Limit theorem helps to approximate distributions using (a) Normal distribution (b) Binomial distribution (c) Poisson distribution (d) Uniform distribution	K2	CO5

Cont...

SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	An urn contains 6 white, 4 red and 9 black balls. If 3 balls are drawn at random, find the probability that: (i) two of the balls drawn are white (ii) one is each colour (iii) none is red.	K2	CO1
		(OR)		
	11.b.	State the applications of Bayes theorem.		
2	12.a.	Sketch the properties of distribution function.	K3	CO2
		(OR)		
	12.b.	Show that (i) If X and Y are random variables, then $E(X + Y) = E(X) + E(Y)$ (ii) If X and Y are independent random variables, then $E(XY) = E(X) \cdot E(Y)$		
3	13.a.	Define (i) Marginal probability distribution (ii) Conditional probability distribution (iii) Joint probability function	K4	CO3
		(OR)		
	13.b.	Given two variates X_1 and X_2 with joint density function $f(x_1, x_2)$, prove that the conditional mean X_2 (given X_1) coincides with (unconditional) mean only if X_1 and X_2 are independent.		
4	14.a.	Describe moment generating function and characteristic function.	K3	CO4
		(OR)		
	14.b.	Four tickets are drawn, one at a time with replacement, from a set of ten tickets numbered respectively, 1, 2, ..., 10, in such a way that at each draw, each ticket is equally likely to be selected. What is the probability that the total of the numbers on the four drawn tickets is 20?		
5	15.a.	State and prove Bernoulli's weak law of large numbers.	K2	CO5
		(OR)		
	15.b.	Examine whether the weak law of large numbers holds for the sequence (X_k) of independent random variables defined as follows: $P(X_k = \pm 2^k) = 2^{-(2k+1)}, P(X_k = 0) = 1 - 2^{-2k}$		

Cont...

SECTION -C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks

(3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO																				
1	16	Suppose one person is selected at random from a group of 100 persons known to confirm the following patterns of political views. <table border="1"> <thead> <tr> <th></th><th>Communist</th><th>Socialist</th><th>Democrat</th><th>Total</th></tr> </thead> <tbody> <tr> <td>Men</td><td>15</td><td>25</td><td>10</td><td>50</td></tr> <tr> <td>Women</td><td>20</td><td>15</td><td>15</td><td>50</td></tr> <tr> <td>Total</td><td>35</td><td>40</td><td>25</td><td>100</td></tr> </tbody> </table> What is the probability that the man selected is a communist?		Communist	Socialist	Democrat	Total	Men	15	25	10	50	Women	20	15	15	50	Total	35	40	25	100	K2	CO1
	Communist	Socialist	Democrat	Total																				
Men	15	25	10	50																				
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2	17	Find the mean and variance of the probability function $p(x) = \begin{cases} \frac{2}{3} \left(\frac{1}{3}\right)^{x-1} & \text{for } x = 1, 2, 3 \\ 0 & \text{otherwise} \end{cases}$	K3	CO2																				
3	18	The joint density function of a bivariate distribution is given by $f(x, y) = 4xye^{-(x^2+y^2)}, x \geq 0, y \geq 0$ Find the marginal and conditional probability density functions. Are x and y independent?	K4	CO3																				
4	19	The probability density function of the random variable X follows the probability law: $p(x) = \frac{1}{2\theta} \exp\left(-\frac{ x - \theta }{\theta}\right), -\infty < x < \infty$ Find M.G.F of X. Hence find E(X) and V(X).	K3	CO4																				
5	20	State and prove weak law of large numbers.	K2	CO5																				

Z-Z-Z END

