PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2026
(Third Semester)

Branch - **MATHEMATICS WITH COMPUTER APPLICATIONS**

DISCRETE MATHEMATICS AND GRAPH THEORY

Time: Three Hours　　　　Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks　　　　(10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	Find that let s and t be statements that contain no logical connectives other than $\vee$ and $\wedge$. If $s \Leftrightarrow t$, then $s^d \Leftrightarrow t^d$. a) Principle of Duality　b) Principle of logical connectives c) Principle of linearity　d) Principle of primitive statements	K1	CO1
	2	Classify the Universally quantified statement $\forall x[p(x) \rightarrow q(x)]$to be $\forall x[\neg q(x) \rightarrow \neg p(x)]$ is said to be ________. a) Inverse　b) Contrapositive c) Converse　d) Universe	K2	CO1
2	3	Find general of the number of compositions of a positive integer n is the coefficient of x^n in $f(x)$ is ________. a)2^n　b)2^{n+1}　c) 2^{n-1}　d)2^{n+2}	K1	CO2
	4	Interpret the value of the expansion $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$ is ________. a) $\frac{e^x+e^{-2x}}{2}$　b)$\frac{e^x-e^{-2x}}{2}$　c)$\frac{e^x-e^{-x}}{2}$　d)$\frac{e^x+e^{-x}}{2}$	K2	CO2
3	5	Find the solution of the recurrence relation $a_n = 7a_{n-1}$, where $n \geq 1$ and $a_2 = 98$. a) $a_n = 2(7)^n$　b)$a_n = 2(7)^{n-1}$ c)$a_n = 2(7)^{n+1}$　d) $a_n = 2(7)^{n/2}$	K1	CO3
	6	Interpret the recurrence relation of the following sequence $8, \frac{24}{7}, \frac{72}{49}, \frac{216}{343}, \ldots$. a) $a_n = \frac{2}{7}a_{n-1}$　b)$a_n = \frac{3}{7}a_{n-1}$ c)$a_n = \frac{4}{7}a_{n-1}$　d)$a_n = \frac{5}{7}a_{n-1}$	K2	CO3
4	7	Let $G = (V, E)$ be an undirected graph, if there is a path between any two distinct vertices of G. Then the graph is said to be a ________ graph. a) Digraph　b) Bipartite c) Petersen　d) Connected	K1	CO4
	8	Classify the chromatic number of the Herschel graph and Petersen graph from the following: a) 3 & 2　b) 3 & 1　c) 2 & 3　d) 2 & 1	K2	CO4
5	9	How many edges must be removed from a connected graph with n vertices and m edges to produce a spanning tree? a) $m + n - 1$　b) $m - n + 1$ c) $n - m + 1$　d) $n - m - 1$	K1	CO5
	10	A tree with n vertices that it can attain is a complete binary tree, then illustrate the maximum height of a tree: a) $\frac{n+1}{2}$ with n odd　b)$\frac{n+1}{2}$ with n even c) $\frac{n-1}{2}$ with n even　d)$\frac{n-1}{2}$ with n odd	K2	CO5

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SECTION - B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Construct the truth tables for Distributive law of primitive statements p, q and r.	K3	CO1
		(OR)		
	11.b.	Construct the Disjunctive Normal Form for (i) $f: B^3 \rightarrow B$,where $f(x,y,z) = xy + \bar{x}z$. (ii)$g(w,x,y,z) = wx\bar{y} + wy\bar{z} + xy$.		
2	12.a.	Solve the following : (i)The coefficient of x^{15} in $f(x) = (x^2 + x^3 + x^4 + \cdots)^4$ (ii) The coefficient of x^8 in $\frac{1}{(x-3)(x-2)^2}$.	K3	CO2
		(OR)		
	12.b.	Solve the equation and determine the number if integral solutions of $x_1 + x_2 + x_3 + x_4 = 18$ subject to $1 \le x_1 \le 5, -2 \le x_2 \le 4$, $0 \le x_3 \le 5, 3 \le x_4 \le 9$.		
3	13.a.	Simplify the recurrence relation $2a_n = 7a_{n-1} - 3a_{n-2};\ a_0 = 2, a_1 = 5.$	K4	CO3
		(OR)		
	13.b.	Simplify the recurrence relation $a_{n+2} - 8a_{n+1} + 16a_n = 8(5)^n + 6(4)^n$; where $n \ge 0$ and $a_0 = 12, a_1 = 5$.		
4	14.a.	Let $G = (V, E)$ be a connected planar graph or multigraph with $\|V\| = v$ and $\|E\| = e$. Let r be the number of regions in the plane determined by a planar embedding of G; one of these regions has infinite area is called the infinite region. Then show that if $v - e + r = 2$.	K2	CO4
		(OR)		
	14.b.	(i) Explain Chromatic number and Chromatic polynomial of undirected graph. (ii) Show that Decomposition theorem for chromatic polynomials of a connected graph.		
5	15.a.	Analyze the following statements are equivalent for a loop-free undirected graph$G = (V, E)$. (i) G is a tree. (ii) $G$is connected, but the removal of any edge from G disconnects G into two subgraphs that are trees. (iii) Gcontains no cycles and $\|V\| = \|E\| + 1$. (iv) $G$is connectedand $\|V\| = \|E\| + 1$.	K4	CO5
		(OR)		
	15.b.	If T is an optimal tree for the n weights $w_1 \le w_2 \le \cdots w_n$,then analyze that there exists an optimal tree T' in which the leaves of weights w_1 and w_2 are siblings at the maximal level (inT') .		

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SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Let p, q, r denote the primitive statements given as p: Roger studied. q: Roger plays racketbal. r: Roger passes discrete Mathematics. Now let p_1, p_2, p_3 denote the premises p_1: If Roger studies, then he will pass discrete Mathematics. p_2: If Roger doesn't play racketball, then he'll study. p_3: Roger failed discrete Mathematics. Whether the argument $(p_1 \wedge p_2 \wedge p_3) \rightarrow q$ is valid and examine the truth table for the implications (i)$[(p \rightarrow r) \wedge (\neg q \rightarrow p) \wedge \neg r] \rightarrow q$. (ii) $[p \wedge ((p \rightarrow r) \rightarrow s)] \rightarrow (r \rightarrow s)$	K4	CO1
2	17	Simplify the following: (i) The Constant in $(4x^3 - \frac{5}{x})^{16}$. (ii) The coefficient of x^{60} in $(x^8 + x^9 + x^{10} + \cdots)^7$. (iii) The sequence generated by $(1-4x)^{\frac{-1}{2}}$.	K4	CO2
3	18	The population of Mumbai city is 6,000,000 at the end of the year 2000.The number of immigrants is 20,000n at the end of the year n.The population of the city increases at the rate of 5% per year. Use A recurrence relation to examine the population of the city at the end of 2010.	K4	CO3
4	19	Analyze that, the loop-free undirected graphs with the following incidence matrices are isomorphic? (i) $A_1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (ii) $A_1 = \begin{bmatrix} 0111 \\ 1010 \\ 1100 \\ 1000 \end{bmatrix}, A_2 = \begin{bmatrix} 0101 \\ 1010 \\ 0101 \\ 1010 \end{bmatrix}$ (iii) $A_1 = \begin{bmatrix} 0101 \\ 1011 \\ 0101 \\ 1110 \end{bmatrix}, A_2 = \begin{bmatrix} 0111 \\ 1010 \\ 1101 \\ 1010 \end{bmatrix}$ (iv) $A_1 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$	K4	CO4
5	20	Examine that (i)Let T be an optimal tree for the weights $w_1 + w_2, w_3, \ldots, w_n$, where $w_1 \leq w_2 \leq w_3 \leq \cdots \leq w_n$. At the leaf with weight $w_1 + w_2$ place a (complete) binary tree of height 1 and assign the weights w_1, w_2 to the children (leaves) of this former leaf. The new binary tree T_1 is constructed is then optimal for the weight $w_1, w_2, w_3, \ldots, w_n$. (ii) Let $G = (V, E)$ be a loop free connected undirected graph with $T = (V, E')$ a depth-first spanning tree of G. If r is the root of T, then r is an articulation point of G if and only if r has at least two children in T.	K4	CO5

Z-Z-Z END