

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2026
(Sixth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

COMPLEX ANALYSIS

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks

(10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	If the function $f(z) = z^2$ then it is in the form $f(z) = u(x, y) + i v(x, y)$ is _____. a) $(x^2 + y^2) + i 2xy$ b) $(x^2 - y^2) + i (2xy)$ c) $(x - y)^2 + i (2xy)$ d) $(x + y)^2 + i 2xy$	K1	CO1
	2	The derivative with respect to z of $(2z^2 + i)^5$ is _____. a) $10z(2z^2 + i)^4$ b) $5(2z^2 + i)^4(4z + i)$ c) $20z(2z^2 + i)^4$ d) $20z^2 + 5i^4$	K2	CO1
2	3	The value of $\int_i^{i/2} e^{\pi z} dz$ is _____. a) $e + \frac{1}{e}$ b) 0 c) $\frac{i-1}{\pi}$ d) $\frac{1+i}{\pi}$	K1	CO2
	4	If $f(z)$ is defined by the equation $f(z) = \begin{cases} 1, & \text{when } y < 0 \\ 4y, & \text{when } y > 0 \end{cases}$ and C is the arc from $z = -1 - i$ to $z = 1 + i$ along the curve $y = x^3$, then the value of the integral is _____. a) $2 + 3i$ b) $3 + 2i$ c) $2 - 3i$ d) $3 - 2i$	K2	CO2
3	5	If C is the positively oriented unit circle $ z = 1$, then the value of $\int_C \frac{e^{2z}}{z^4} dz$ is _____. (a) $\frac{7\pi i}{3}$ (b) $\frac{8i}{3}$ (c) $\frac{3\pi i}{3}$ (d) $\frac{8\pi i}{3}$	K1	CO3
	6	If f is entire and bounded in the complex plane, then $f(z)$ is constant throughout the plane. This is known as _____. (a) Fundamental Theorem of Algebra. (b) Maximum Modulus Principle Theorem. (c) Liouville's Theorem. (d) Cauchy's Theorem.	K2	CO3
4	7	If $f(z) = \frac{(\log z)^3}{z^2 + 1}$, then the residue of $f(z)$ at $z = i$ is _____. (a) $-\frac{\pi^3}{16}$ (b) $-\frac{16}{\pi^3}$ (c) $\frac{\pi^3}{16}$ (d) $\frac{3\pi^2}{8}$	K1	CO4
	8	If $f(z) = \frac{z^3 + 2z}{(z-i)^3}$, then it has a pole of order _____. (a) 2 (b) 3 (c) i (d) -3	K2	CO4
5	9	The residue of the function $f(z) = \frac{z}{z^4 + 4}$ is _____. (a) $\frac{i}{8}$ (b) $\frac{1}{4i}$ (c) $\frac{3i}{8}$ (d) $-\frac{i}{8}$	K1	CO5
	10	$\pi i \cdot \sum_{k=1}^n \text{Res}_{z=z_k} f(z) =$ _____. (a) $\int_{-\infty}^{\infty} f(x) dx$ (b) $\int_0^{\infty} f(x) dx$ (c) $\int_{-R}^R f(z) dz$ (d) $\int_0^R f(z) dz$	K2	CO5

Cont...

SECTION - B (35 Marks)Answer **ALL** questions**ALL** questions carry **EQUAL** Marks

(5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	State and prove the Cauchy-Riemann equations.	K3	CO1
	(OR)			
	11.b.	Show that the function $f(z) = \cosh x \cos y + i \sinh x \sin y$ is nowhere analytic.		
2	12.a.	Evaluate $\int_C \frac{z+2}{z} dz$, where C is the upper half of the circle $ z = 2$, from right to left.	K4	CO2
	(OR)			
	12.b.	Apply the Cauchy-Goursat Theorem to evaluate $\int_C \frac{dz}{2z+1}$, when the contour C is $ z = 1$.		
3	13.a.	State and prove the Maximum Modulus Principle Theorem.	K3	CO3
	(OR)			
	13.b.	State and prove the Fundamental Theorem of Algebra.		
4	14.a.	Define: (i) Isolated singular point, (ii) Removable singular point, (iii) Essential singular point.	K4	CO4
	(OR)			
	14.b.	Use the Cauchy's Residue theorem to evaluate $\int_C \frac{5z-2}{z(z-1)} dz$, when C is the circle $ z = 2$.		
5	15.a.	State and prove Riemann's theorem.	K3	CO5
	(OR)			
	15.b.	Prove that a function f has a zero of order m at z_0 if and only if there is a function g , which is analytic and nonzero at z_0 , such that $f(z) = (z - z_0)^m g(z)$.		

SECTION - C (30 Marks)Answer **ANY THREE** questions**ALL** questions carry **EQUAL** Marks

(3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Show that $u(x, y) = y^3 - 3x^2y$ is harmonic in some domain, find its harmonic conjugate $v(x, y)$, and also find the analytic function $f(z)$.	K3	CO1
2	17	State and prove the Cauchy-Goursat Theorem.	K4	CO2
3	18	State and prove Taylor's Theorem.	K4	CO3
4	19	State and prove the Cauchy's Residue Theorem.	K4	CO4
5	20	Show that $\int_0^\infty \frac{x^2}{x^6+1} dx = \frac{\pi}{6}$.	K5	CO2

Z-Z-Z

END