25MCU102 /22MCU102N

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2026
(First Semester)

Branch- **MATHEMATICS WITH COMPUTER APPLICATIONS**

ORDINARY DIFFERENTIAL EQUATIONS AND LAPLACE TRANSFORMS

Time: Three Hours — Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	The differential equation $\frac{dy}{dx} = \frac{y}{x}$ is separable. Its general solution is a) $y = x + C$ b) $y = Cx$ c) $y = Cx^2$ d) $y = \ln x + C$	K3	CO1
	2	The direction field for the equation $\frac{dy}{dx} = x^2 - y^2$ suggests which type of behavior near the origin? a) Saddle point b) Spiral source c) Center d) Node	K3	CO1
2	3	The integrating factor for the linear equation $\frac{dy}{dx} + \frac{2}{x}y = x^3$ a) e^{2x} b) x^{-2} c) x^2 d) None	K3	CO2
	4	For the I.V.P $\frac{dy}{dx} = y + \sin x, y(0) = 1$ the existence and uniqueness theorem guarantees a unique solution because a) $f(x,y)$ is continuous in x only b) $f(x,y)$ and $\frac{\partial f}{\partial y}$ are continuous in a region containing $(0,1)$ c) $f(x,y)$ is differentiable in y only d) $f(x,y)$ is bounded	K3	CO2
3	5	The equation $(xy + 1)dx + x^2dy = 0$ is not exact. Which integrating factor makes it exact? a) $\frac{1}{x}$ b) $\frac{1}{y}$ c) $\frac{1}{xy}$ d) x	K2	CO3
	6	The Bernoulli equation $\frac{dy}{dx} + \frac{2}{x} = x^2y^3$ is transformed into a linear equation by the substitution of ______ a) $u = y^{-2}$ b) $u = y^{-3}$ c) $u = y^2$ d) $u = \frac{1}{y}$	K2	CO3
4	7	*L(sin at)* = ______ a) $\frac{a}{s^2+a^2}$ b) $\frac{s}{s^2+a^2}$ c) $\frac{a}{--a^2}$ d) $\frac{s}{s^2-a^2}$	K2	CO4
	8	The inverse Laplace transform of $\frac{1}{s}$ is a) 1 b) t c) t^2 d) 0	K2	CO4
5	9	$If\ L(f(t) = F(s), then\ L(tf(t)) =$ ____ a) $-F'(s)$ b) $F'(s)$ c) sf(s) d) $sF'(s)$	K3	CO5
	10	The value of the integral $\int_0^\infty e^{-t}\cos t\, dt =$ _____. a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) 2 d) -2	K3	CO5

Cont...

SECTION - B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Suppose that in 1885 the population of a certain country was 50 million and was growing at the rate of 750,000 people per year at that time. Suppose also that in 1940 its population was 100 million and was then growing at the rate of 1 million per year. Assume that this population satisfies the logistic equation. Determine both the limiting population M and the predicted population for the year 2000.	K3	CO1
		(OR)		
	11.b.	Solve the differential equation $\frac{dy}{dx} = (x + y + 3)^2$ using substitution method.		
2	12.a.	Show that the functions $y_1(x) = e^{-3x}$, $y_2(x) = cos\, 2x$ and $y_3(x) = sin\, 2x$ are linearly independent	K3	CO2
		(OR)		
	12.b.	Derive the principle of Superposition for homogeneous equation.		
3	13.a.	Compute the initial value problem $y^{(3)} + 3y'' - 10y' = 0$, $y(0) = 7, y'(0) = 0, y''(0) = 70$.	K2	CO3
		(OR)		
	13.b.	$Solve\ y'' - 4y' + 5y = 0$ for which $y(0) = 1$ and $y'(0) = 5$.		
4	14.a.	Find the inverse Laplace Transform of $G(s) = \frac{1}{s^2(s-a)}$	K2	CO4
		(OR)		
	14.b.	Find the inverse Laplace Transform of $R(s) = \frac{s^2+1}{s^3-2s^2-8s}$		
5	15.a.	Express the convolution of $cos\, t$ and $sin\, t$ i.e., $(cos\, t) * (sin\, t)$	K3	CO5
		(OR)		
	15.b.	Compute $\mathcal{L}^{-1}\left\{tan^{-1}\left(\frac{1}{s}\right)\right\}$.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Solve the initial value problem $\frac{dy}{dx} - y = \frac{11}{8} e^{-x/3}, y(0) = -1$.	K3	CO1
2	17	Express the Wronskians of solutions.	K3	CO2
3	18	Compute the solution of $y'' + 4y = 3x^2$.	K2	CO3
4	19	Consider a mass-and-spring system with $m = \frac{1}{2}$, $k = 17$, and $c = 3$ in mks units. As usual, Let $x(t)$ denote the displacement of the mass m from its equilibrium position. If the mass is set in motion with $x(0) = 3$ and $x'(0) = 1$, find $x(t)$ for the resulting damped free oscillations.	K2	CO4
5	20	A mass that weights 32 $lb(mass\ m = 1\ slug)$ is attached to the free end of a long, light spring that is stretched $1ft$ by a force of $4lb(k = 4\ lb/ft)$. The mass is initially at rest in its equilibrium position. Beginning at time $t = 0$ (seconds), an external force $f(t) = cos\ 2t$ is applied to the mass, but at time $t = 2\pi$ this force is turned off (abruptly discontinued) and the mass is allowed to continue its motion unimpeded. Find the resulting position function $x(t)$ of the mass.	K3	CO5

Z-Z-Z END