

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2026
(Third Semester)

Branch – MATHEMATICS WITH COMPUTER APPLICATIONS

ADVANCED MATHEMATICAL STATISTICS - I

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	Which of the following is a certain event? (i) Rolling a 7 on a dice (ii) Getting an even number on a dice (iii) Getting a number between 1 and 6 on a dice (iv) Drawing a red card from a standard deck	K1	CO1
	2	The addition law of probability for two events A and B is: (i) $P(A \cup B) = P(A) + P(B)$ (ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (iii) $P(A \cup B) = P(A) - P(B)$ (iv) $P(A \cup B) = P(A) \times P(B)$	K2	CO1
2	3	The cumulative distribution function (CDF) of a random variable X, F(x), satisfies: (i) F(x) is non-decreasing (ii) F(x) can be negative (iii) F(x) always equals 1 for all x (iv) F(x) decreases with x	K1	CO2
	4	If X and Y are two random variables, the marginal distribution of X is: (i) The joint probability of X and Y (ii) The distribution of X ignoring Y (iii) The conditional distribution of X given Y (iv) The sum of probabilities of X and Y	K2	CO2
3	5	The expected value of a random variable X is defined as: (i) Sum of all probabilities (ii) Weighted average of all possible values of X (iii) Square root of variance (iv) Maximum value of X	K1	CO3
	6	If X and Y are independent random variables, the expectation of their sum is: (i) $E(X + Y) = E(X) \times E(Y)$ (ii) $E(X + Y) = E(X) + E(Y)$ (iii) $E(X + Y) = E(X) - E(Y)$ (iv) $E(X + Y) = 0$	K2	CO3
4	7	The moment generating function (mgf) of a random variable X is defined as: (i) $M_X(t) = E(X)$ (ii) $M_X(t) = E(e^{tX})$ (iii) $M_X(t) = \text{Var}(X)$ (iv) $M_X(t) = P(X \leq t)$	K1	CO4
	8	The nth moment of a random variable X about the origin is: (i) $E[(X - \mu)^n]$ (ii) $E[X^n]$ (iii) $E[(X + \mu)^n]$ (iv) $E[X/n]$	K2	CO4
5	9	The binomial distribution is used when: (i) There are two possible outcomes for each trial (ii) The number of trials is infinite (iii) Events are dependent (iv) Data is continuous	K1	CO5
	10	For a normal distribution $N(\mu, \sigma^2)$, which of the following is true? (i) Mean = μ , Variance = σ^2 (ii) Mean = σ , Variance = μ^2 (iii) Mean = μ^2 , Variance = σ (iv) Mean = 0, Variance = 1 always	K2	CO5

Cont...

Module No.	Question No.	Question	K Level	CO
1	11.a.	Two dice are thrown together. Find the probability of: (a) a sum of 8 (b) doubles	K2	CO1
		(OR)		
	11.b.	vents A and B are such that $P(A) = 0.5$, $P(B) = 0.6$, and $P(A \cap B) = 0.3$. Find: (a) $P(A \cup B)$ (b) $P(A B)$ (c) $P(B A)$		
2	12.a.	A random variable X takes values 1, 2, 3, 4 with probabilities 0.1, 0.3, 0.4, 0.2. Find: (a) the mean and variance of X (b) the cumulative distribution function	K2	CO1
		(OR)		
	12.b.	The probability density function of a continuous random variable X is given by $f(x) = 2x$, $0 \leq x \leq 1$. Find: (a) the probability that $X \leq 0.5$ (b) the mean and variance of X		
3	13.a.	A random variable X takes values 1, 2, 3 with probabilities 0.2, 0.5, 0.3. Find: (a) $E(X)$ (b) $E(3X + 2)$	K6	CO2
		(OR)		
	13.b.	For two random variables X and Y, $E(X) = 4$, $E(Y) = 5$, $\text{Var}(X) = 2$, $\text{Var}(Y) = 3$, $\text{Cov}(X, Y) = 1$. Find: (a) $E(2X + 3Y)$ (b) $\text{Var}(2X + 3Y)$		
4	14.a.	how that if X and Y are independent random variables, $M_{X+Y}(t) = M_X(t) \times M_Y(t)$. Illustrate with X and Y taking values 0 and 1 with equal probabilities.	K4	CO3
		(OR)		
	14.b.	A discrete random variable X takes values 0, 1, 2 with probabilities 0.2, 0.5, 0.3. Find: (a) the moment generating function $M_X(t)$ (b) $E(X)$ and $\text{Var}(X)$ using the mgf		
5	15.a.	A coin is tossed 5 times. Find the probability of getting exactly 3 heads using the binomial distribution.	K5	CO4
		(OR)		
	15.b.	A factory produces items, and the number of defective items per day follows a Poisson distribution with mean 2. Find the probability of: (a) No defective items (b) Exactly 3 defective items		

SECTION -C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	A diagnostic test for a disease has: $P(\text{Test} + \text{Disease}) = 0.95$, $P(\text{Test} - \text{No disease}) = 0.9$, and the disease prevalence = 0.01. Using Bayes' theorem, find the probability that a person who tested positive actually has the disease.	K2	CO1
2	17	The joint probability density function of X and Y is given by $f(x, y) = 6xy$, $0 \leq x \leq 1$, $0 \leq y \leq 1$. Find: (a) the marginal density functions of X and Y (b) the probability $P(X + Y \leq 1)$ (c) $E(X)$ and $E(Y)$	K6	CO2
3	18	A random variable X takes values 1, 2, 3, 4 with probabilities 0.1, 0.2, 0.4, 0.3. Find: (a) $E(X)$ (b) $E(2X + 3)$ (c) $E(X^2)$ and $\text{Var}(X)$	K5	CO4
4	19	Let X be a random variable with characteristic function $\phi_X(t) = 0.25 + 0.5 e^{it} + 0.25 e^{2it}$. (a) Verify $\phi_X(0) = 1$ (b) Find $E(X)$ and $\text{Var}(X)$ using $\phi_X(t)$ (c) Comment on the distribution of X	K4	CO3
5	20	A random variable X follows a normal distribution $N(\mu = 50, \sigma = 5)$. (a) Find the probability that X lies between 45 and 55 (b) Find X such that $P(X \leq x) = 0.975$ (c) Verify the mean and variance using the mgf	K4	CO3