**PSG COLLEGE OF ARTS & SCIENCE**
(AUTONOMOUS)

**BSc DEGREE EXAMINATION MAY 2026**
(Fifth Semester)

Branch - **MATHEMATICS**

# REAL ANALYSIS

Time: Three Hours | Maximum: 75 Marks

## SECTION-A (10 Marks)

Answer **ALL** questions
**ALL** questions carry **EQUAL** marks (10 × 1 = 10)

| Module No. | Question No. | Question | K Level | CO |
|---|---|---|---|---|
| 1 | 1 | Let $E_1$ be the set of all $r \varepsilon Q$ with $r < 0$. Let $E_2$ be the set of all $r \varepsilon Q$ with $r \le 0$. Then<br>(i) Sup $E_1$ = Sup $E_2 = 0$ (ii) Sup $E_1$ < Sup $E_2$<br>(iii) $Sup\ E_1 > Sup\ E_2$ (iv) $Sup\ E_1 = Sup\ E_2$ | K1 | CO1 |
| | 2 | If z is a complex number, its absolute value $\|z\|$ is the no negative square root of $z\bar{z}$; that is $\|z\|$= -----<br>(i) $(z\bar{z})$ (ii) $(z\bar{z})^2$ (iii) $(z\bar{z})^{\frac{1}{2}}$ (iv) $(z\bar{z})^3$ | K2 | CO1 |
| 2 | 3 | If $E \subset B, f^{-1}(E)$ denotes the set of all $x \in A$ such that $f(x) \in E$. We call $f^{-1}(E)$ the ------- of E under $f$.<br>(i) image (ii) inverse image<br>(iii) mapping (iv) function | K1 | CO2 |
| | 4 | Every neighborhood is an --------<br>(i) open set (ii) closed set<br>(iii) limit point (iv) interior point | K2 | CO2 |
| 3 | 5 | Every interval [a,b] (a<b) is --------<br>(i) finite (ii) countable<br>(iii) bounded (iv) uncountable | K1 | CO3 |
| | 6 | The interval [0,1] and the segment (1,2) are ---------<br>(i) not connected (ii) connected<br>(iii) not separated (iv) separated | K2 | CO3 |
| 4 | 7 | If $\bar{E}$ is the closure of a set E in a metric space X, then $diam\ \bar{E} = -----$<br>(i) E (ii) diam E (iii) $diam\ E_n$ (iv) $diam\ K_n$ | K1 | CO4 |
| | 8 | A metric space in which every Cauchy sequence converges is said to be ----------<br>(i) complete (ii) increasing sequence<br>(iii) decreasing sequence (iv) monotonic sequence | K2 | CO4 |
| 5 | 9 | Let $s_n = (-1^n)/[1+(1/n)]$. Then $\limsup_{n\to\infty} s_n =$ ------<br>(i) $s$ (ii) 0 (iii) -1 (iv) 1 | K1 | CO5 |
| | 10 | If $\sum a_n$ converges, then $\lim_{n\to\infty} a_n$ = ---------<br>(i) 1 (ii) -1/n (iii) 0 (iv) 1/n | K2 | CO5 |

**Cont...**

## SECTION - B (35 Marks)

Answer **ALL** questions

**ALL** questions carry **EQUAL** Marks (5 × 7 = 35)

| Module No. | Question No. | Question | K Level | CO |
|---|---|---|---|---|
| 1 | 11.a. | Prove that for every real $x > 0$ and every integer $n > 0$ there is one and only one positive real y such that $y^n = x$. | K2 | CO1 |
| | | (OR) | | |
| | 11.b. | If $a_1,...,a_n$ and $b_1,...,b_n$ are complex numbers, prove that $\left\lvert\sum a_j \overline{b_j}\right\rvert^2 \le \sum_{j=1}^{n} \lvert a_j\rvert^2 \sum_{j=1}^{n} \lvert b_j\rvert^2$. | | |
| 2 | 12.a. | Prove that every infinite subset of a countable set A is countable. | K3 | CO2 |
| | | (OR) | | |
| | 12.b. | Prove that<br>i) For any collection $\{F_\alpha\}$ of closed sets $\bigcap_\alpha F_\alpha$ is closed.<br>ii) For any finite collection $F_1,...,F_n$ of closed sets $\bigcup_{i=1}^{n} F_i$ is closed. | | |
| 3 | 13.a. | Suppose $K \subset Y \subset X$. Prove that K is compact relative to X if and only if K is compact relative to Y. | K3 | CO3 |
| | | (OR) | | |
| | 13.b. | Let P be a nonempty perfect set in $R^k$. Prove that P is uncountable. | | |
| 4 | 14.a. | Prove that the sub sequential limits of a sequence $\{p_n\}$ in a metric space X form a closed subset of X. | K3 | CO4 |
| | | (OR) | | |
| | 14.b. | Suppose $\{s_n\}$ is monotonic. Prove that $\{s_n\}$ converges if and only if it is bounded. | | |
| 5 | 15.a. | Prove that e is irrational. | K3 | CO5 |
| | | (OR) | | |
| | 15.b. | Suppose<br>i) the partial sums $A_n$ of $\sum a_n$ form a bounded sequence.<br>ii) $b_0 \ge b_1 \ge ...$<br>iii) $\lim_{n\to\infty} b_n$ . Prove that $\sum a_n b_n$ converges. | | |

## SECTION -C (30 Marks)

Answer **ANY THREE** questions

**ALL** questions carry **EQUAL** Marks (3 × 10 = 30)

| Module No. | Question No. | Question | K Level | CO |
|---|---|---|---|---|
| 1 | 16 | i) If $x \in R, y \in R$ and x>0, prove that there is a positive integer n such that $nx > y$.<br>ii) If $x \in R, y \in R$ ,and x<y, prove that there exist a $p \in Q$ such that $x < p < y$. | K2 | CO1 |
| 2 | 17 | If X is a metric space and $E \subset X$, prove that<br>i) $\overline{E}$ is closed,<br>ii) $E = \overline{E}$ if and only if E is closed,<br>iii) $\overline{E} \subset F_\alpha$ for every closed set $F_\alpha \subset X$ such that $E \subset F_\alpha$. | K2 | CO2 |
| 3 | 18 | If a set E in $R^k$ has one of the following three properties, prove that it has the other two<br>i) E is closed and bounded.<br>ii) E is compact.<br>iii) Every infinite subset of E has a limit point in E. | K3 | CO3 |
| 4 | 19 | Suppose $\{s_n\},\{t_n\}$ are complex sequences and $\lim\limits_{n\to\infty} t_n = t$. Prove that<br>*i)* $\lim\limits_{n\to\infty}(s_n + t_n) = s + t$;<br>*ii)* $\lim\limits_{n\to\infty} cs_n = cs, \lim\limits_{n\to\infty}(c + s_n) = c + s$ *for any number* $c$;<br>*iii)* $\lim\limits_{n\to\infty} s_n t_n \; st$;<br>*iv)* $\lim\limits_{n\to\infty} \frac{1}{s_n} = \frac{1}{s}$, *provided* $s_n \neq 0$ | K4 | CO4 |
| 5 | 20 | Prove that $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e.$ | K4 | CO5 |

Z-Z-Z END