PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2026
(Third Semester)

Branch - **MATHEMATICS**

PARTIAL DIFFERENTIAL EQUATIONS AND FOURIER TRANSFORMS

Time: Three Hours | Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks (10 × 1 = 10)

Question No	Question	K Level	CO
1	Which is the non- linear PDE of first order? a) $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=nz$ b) $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=z^2$ c) $P(z)\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=0$ d) $\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2=1$	K1	CO1
2	What is the type of linear PDE when $F(u,v)=0$? a) $Pp+Qq=R$ b) $Pp-Qq=R$ c) $P/Q=R$ d) $PQ=R$	K2	CO1
3	The operator $F(D,D')u=(D^2-D')$ is _____. a) reducible b) irreducible c) Unitary d) self-adjoint	K2	CO2
4	Which of the following is linear three-dimensional wave equation? a) $u_t=k(u_{xx}+u_{yy}+u_{zz})$ b) $u_{xx}+u_{yy}+u_{zz}=0$ c) $u_{tt}=c^2(u_{xx}+u_{yy}+u_{zz})$ d) $u_t+uu_x=\mu u_{xx}$	K1	CO2
5	The Fourier cosine series is_____ a) $a_0+\sum_{n=1}^{\infty}a_n\cos nx$ b) $a_0+\sum_{n=1}^{\infty}a_n\sin nx$ c) $\sum_{n=1}^{\infty}a_n\cos nx$ d) $\sum_{n=1}^{\infty}a_n\sin nx$	K1	CO3
6	If the constants in a trigonometric series is converges, then its sum will be a function of period_____ a) π b) $-\pi$ c) 2π d) -2π	K2	CO3
7	The function f(x)= $\lim_{L\to\infty} f_L(x)$ is_____ a) differentiable b) integrable c) absolute differentiable d) absolute integrable	K2	CO4
8	The Fourier sine integral is_____ a) $f(x)=\int_0^{\infty}B(w)\sin wxdw$ b) $f(x)=-\int_0^{\infty}B(w)\sin wxdw$ c) $f(x)=\int_0^{\infty}A(w)\cos wxdw$ d) $f(x)=-\int_0^{\infty}A(w)\cos wxdw$	K1	CO4
9	Which of the following equation represents the heat equation? a) $\frac{\partial u}{\partial t}=c^2\nabla^2u$ b) $\frac{\partial^2u}{\partial t^2}=c^2\nabla^2u$ c) $\frac{\partial u}{\partial t}=\frac{1}{c^2}\nabla^2u$ d) $\frac{\partial^2u}{\partial t^2}=\frac{1}{c^2}\nabla^2u$	K1	CO5
10	What is the value of the thermal conductivity k when the material is homogeneous and the temperature is non-extreme? a) Constant b) Zero c) Variable d) Infinite	K2	CO5

Cont...

SECTION – B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks (5 × 7 = 35)

Question No	Question	K Level	CO
11.a.	Find the partial differential equation by eliminating the arbitrary function from $f(x+y+z, x^2+y^2+z^2)=0$	K2	CO2
	(OR)		
11.b.	Find the complete integral of the PDE: $z^2 = pqxy$.		
12.a.	Solve the equation $(D^2+3DD'+2D'^2)u = x+y$	K3	CO3
	(OR)		
12.b.	Solve the following PDE: $(x^2D^2+2xyDD'+y^2D'^2)u = x^2y^2$		
13.a.	State and prove orthogonality of the Trigonometric system.	K3	CO3
	(OR)		
13.b.	Find the Fourier series of the function $f(x)=\begin{cases}0 & if \quad -2<x<-1\\ k & if \quad -1<x<1\\ 0 & if \quad 1<x<2\end{cases}$ $p=2L=4, \quad L=2$		
14.a.	Find the Fourier transform of $f(x)=1$ if $\lvert x\rvert<1$ and $f(x)=0$ otherwise.	K4	CO4
	(OR)		
14.b.	State and prove Fourier transform of the derivatives of $f(x)$.		
15.a.	Find the temperature $u(x,t)$ in a laterally insulated copper bar 80 cm long if the initial temperature is $100\sin(\pi x/80)^\circ C$ and the ends are kept at $0^\circ C$. How long will it take for the maximum temperature in the bar to drop to $50^\circ C$?	K4	CO4
	(OR)		
15.b.	Solve the heat problem $f(x)=\begin{cases}U_0 = const & if \quad \lvert x\rvert<1\\ 0 & if \quad \lvert x\rvert>1\end{cases}$ by the method of convolution.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** marks (3 × 10 = 30)

Question No	Question	K Level	CO
16	Prove that the general solution of the linear PDE $Pp+Qq=R$ can be written in the form $F(u,v)=0$.	K4	CO4
17	Reduce the following equation $u_{xx}+2u_{xy}+4u_{yy}+2u_x+3u_y=0$ to canonical form and hence solve it.	K4	CO4
18	A sinusoidal voltage $E\sin\omega t$, where t is time, is passed through a half-wave rectifier that clips the negative portion of the wave. Find the Fourier series of the resulting periodic function $U(t)=\begin{cases}0 & if -L<t<0,\\ E\sin\omega t & if \quad 0<t<L\end{cases}$ $p=2L=\frac{2\pi}{\omega}, \quad L=\frac{\pi}{\omega}.$	K4	CO4
19	State and Prove Convolution theorem.	K4	CO4
20	Find the temperature in an Infinite bar.	K4	CO4

Z-Z-Z END