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22MAU410N

PSG COLLEGE OF ARTS & SCIENCE

(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2026

(Fourth Semester)

Branch – **MATHEMATICS**

MODERN ALGEBRA

Time: Three Hours   Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks $(10 \times 1 = 10)$

Module No.	Question No.	Question	K Level	CO
1	1	If G is a finite group and $a \in G$, then $a^{o(G)} =$ __. a) 0 b) 1 c) e d) ∞	K1	CO1
	2	If p is a prime number and 'a' is any integer, then $a^p \equiv$ _mod p. a) a b) 1 c) 0 d) p	K2	CO1
2	3	The kernel of the homomorphism f: $(R^*, .) \to (R^+, .)$ defined by f(x) = |x| is ____ a) {1} b) {-1} c) {0} d) {1,-1}	K1	CO1
	4	The Kernel of the homomorphism f: $(Z, +) \to (Z, +)$ defined by f(x) = 2x is a) Z b) {1/2} c) {1} d) {0}	K2	CO2
3	5	Suppose (2, 5, 8, 4) and (3, 6) are the two permutation groups that form cycles. What type of permutation is this? a) odd b) even c) acyclic d) prime	K1	CO2
	6	The number of automorphisms of a cyclic group of order n is ____ a) $\varphi(n)$ b) n c) n^2 d) 1	K2	CO2
4	7	Which of the following is not a ring ? a) (Z, +, .) b) (Q, +, .) c) (R, +, .) d) (R, ., +)	K1	CO3
	8	A finite integral domain is a______. a) a Field b) not field c) Euclindean ring d) Integral domain	K2	CO3
5	9	Any ordered integral domain is of characteristic ___ a) 1 b) 0 c) infinite d) prime	K1	CO4
	10	If R is a finite commutative ring with identity, then every prime ideal if R is a) prime ideal of R b) not a prime ideal c) maximal ideal d) not a maximal ideal	K2	CO4

Cont…

SECTION - B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	If G is a finite group and $a \in G$, then show that $o(a)/o(G)$.	K2	CO1
		(OR)		
	11.b.	If H is a non empty finite subset of a group G and H is closed under multiplication, then examine that H is a subgroup of G.		
2	12.a.	Analyze the statement N is a normal subgroup of G if and only if $gNg^{-1} = N$ for every $g \in G$.	K4	CO1
		(OR)		
	12.b.	If ϕ is a homomorphism of G into $\bar{G}$ with kernel K, then examine that K is a normal subgroup of G.		
3	13.a.	If G is a group, then show that $\mathcal{A}(G)$, the set of automorphisms of G, is also a group.	K3	CO2
		(OR)		
	13.b.	Prove that every permutation is the product of its cycles.		
4	14.a.	If $\phi$is a homomorphism of R into R', then examine the following: i) $\phi(0) = 0$. ii) $\phi(-a) = -\phi(a)\ for\ every\ a \in R$.	K5	CO3
		(OR)		
	14.b.	If U is an ideal of the ring R, then prove that R/U is a ring and is a homomorphic image of R.		
5	15.a.	Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Then Prove that R is a field.	K5	CO4
		(OR)		
	15.b.	Show that a Euclidean ring possesses a unit element.		

SECTION –C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Show that the relation $a \equiv b\ mod\ H$ is an equivalence relation.	K2	CO1
2	17	State and prove Cauchy's theorem for Abelian Groups.	K3	CO2
3	18	State and prove Cayley theorem.	K3	CO2
4	19	If R is a ring, then for all $a, b \in R$ i) a0 =0 a=0 ii) a(-b) = (-a)b = - (ab) iii) (-a)(-b) = ab. If, in addition, R has a unit element 1, then Prove that the following: iv) (-1)a = - a v) (-1)(-1) = 1.	K5	CO3
5	20	Prove that Every integral domain can be imbedded in a field.	K4	CO4

Z-Z-Z END