PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2025
(First Semester)

Branch - **STATISTICS**

REAL ANALYSIS AND LINEAR ALGEBRA

Time: Three Hours — Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 × 1 = 10)

Question No.	Question	K Level	CO
1	The supremum of the set {1 – 1/n : n ∈ N} is: (a) 0 (b) 1 (c) ∞ (d) –1	K1	CO1
2	Bolzano–Weierstrass theorem states that every bounded sequence has: (a) Limit (b) Subsequence (c) Convergent subsequence (d) Divergent subsequence	K2	CO1
3	The derivative of $f(x) = x^3$ at $x = 2$ is: (a) 6 (b) 8 (c) 12 (d) 4	K1	CO2
4	A function is uniformly continuous on a closed interval [a,b] if it is: (a) Continuous (b) Differentiable (c) Bounded (d) Continuous on [a,b]	K2	CO2
5	The Riemann integral of f(x)=1 on [0,1] is: (a) 0 (b) ½ (c) 1 (d) ∞	K1	CO3
6	If f is integrable on [a,b], then the Fundamental Theorem of Calculus relates: (a) Sum and product (b) Differentiation and Integration (c) Limit and continuity (d) None	K2	CO3
7	The dimension of R^3 as a vector space is: (a) 1 (b) 2 (c) 3 (d) ∞	K1	CO4
8	The Gram-Schmidt process is used to: (a) Find determinant (b) Orthogonalize vectors (c) Find eigenvalues (d) Invert matrix	K2	CO4
9	If A is a 2X2 matrix with determinant ≠0, then A is: (a) Not invertible (b) Invertible (c) Singular (d) Nilpotent	K1	CO5
10	A quadratic form is positive definite if: (a) All eigenvalues positive (b) All eigenvalues negative (c) At least one eigenvalue zero (d) None	K2	CO5

SECTION - B (35 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Question No.	Question	K Level	CO
11.a.	State and prove the Bolzano–Weierstrass theorem.	K2	CO1
	(OR)		
11.b.	Test the convergence of the series $\sum (-1)^n / n$.		

Cont...

12.a.	State and prove Mean Value Theorem for differentiable functions.	K3	CO2
(OR)			
12.b.	Find local maxima and minima of $f(x,y) = x^2 + y^2 - 2x - 4y + 5$		
13.a.	Define Riemann integral. Show that every continuous function on [a,b] is Riemann integrable.	K3	CO3
(OR)			
13.b.	Evaluate $\int_0^4 x^2\, dx$ using definition of Riemann integral.		
14.a.	Apply Gram-Schmidt process to orthogonalize the set {(1,1,0), (1,0,1)} in R^3.	K4	CO4
(OR)			
14.b.	Define inner product space. Show that $(x,y) = x_1y_1+x_2y_2+.....+x_ny_n$ is inner product on R_n.		
15.a.	Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$.	K5	CO5
(OR)			
15.b.	Explain how the generalized inverse of a matrix can be used to obtain the solution of a system of linear equations $AX = B$.		

SECTION –C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Question No.	Question	K Level	CO
16	Define absolute and conditional convergence. Discuss tests for convergence of series with examples.	K5	CO1
17	State and prove Taylor's theorem with remainder. Illustrate with an example.	K5	CO2
18	Explain Riemann-Stieltjes integral.	K5	CO3
19	Define linear transformation. Find the matrix linear transformation $T:R^2 \rightarrow R^2$ given by $T(x,y) = (X+2y, 3x+y)$.	K5	CO4
20	Verify the Caley – Hamilton theorem for $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$. Hence find its inverse.	K5	CO5

Z-Z-Z END