PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2025
(First Semester)

Branch - **MATHEMATICS**

MATHEMATICAL STATISTICS

Time: Three Hours | Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	A random variable is a function that assigns a ______ to each outcome of a random experiment. a) probability b) real number c) sample space d) distribution	K1	CO1
	2	The probability mass function (PMF) is associated with which type of random variable? a) Continuous b) Both discrete and continuous c) Discrete d) None of these	K2	CO1
2	3	The expectation of a random variable X is denoted by: a) V(*X*) b) C(*X*) c) E(*X*) d) M(*X*)	K1	CO1
	4	The covariance between X and Y is given by a) E(*XY*)-*E*(*X*)*E*(*Y*) b) E(XY) c) E(X)+E(Y) d) E(X)-E(Y)	K2	CO2
3	5	If the MGF of a random variable exists in an open interval containing 0, it uniquely determines: (a) The mean only (b) The variance only (c) The distribution of the random variable (d) None of these	K1	CO1
	6	Which among the following is not a property of the cumulant generating function? (a) Cumulants are additive for independent random variables (b) The first cumulant equals the mean (c) The second cumulant equals the variance (d) The third cumulant always equals zero	K2	CO2
4	7	The mean of the Binomial distribution B(n,p) is a) n^2p b) np c) $p(1-p)$ d) $\sqrt{np(1-p)}$	K1	CO1
	8	The fourth central moment of a Poisson distribution with parameter λ is a) λ b) $3\lambda^2+\lambda$ c) λ^4 d) $\sqrt{\lambda}$	K2	CO2
5	9	The Chi-square distribution is a special case of: a) Normal distribution b) F-distribution c) Gamma distribution d) t-distribution	K1	CO1
	10	The Chi-square distribution is always: (a) Symmetric (b) Positively skewed (c) Negatively skewed (d) Normal for any n	K2	CO2

SECTION - B (35 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Let X be a continuous random variable with p.d.f $f(x)=kx(1-x)$, with range space $R:\{x:0\le x\le 1\}$. Find k and determine a number b such that $P(x\le b)=P(x\ge b)$.	K3	CO3
		(OR)		
	11.b.	The joint probability density function of two dimensional random variable X and Y is given by $f(x,y)=\begin{cases}2, 0\le x\le y\le 1\\ 0, otherwise\end{cases}$ Find the distribution of $U=X+Y$.		

Cont...

<table>
<tr><td rowspan="3">2</td><td>12.a.</td><td>The probability function of two dimensional random variables X and Y is shown below:
<table>
<tr><th>X / Y</th><th>0</th><th>1</th><th>2</th><th>3</th><th>h(y)</th></tr>
<tr><td>1</td><td>1/18</td><td>3/18</td><td>1/18</td><td>1/18</td><td>6/18</td></tr>
<tr><td>2</td><td>2/18</td><td>2/18</td><td>1/18</td><td>2/18</td><td>7/18</td></tr>
<tr><td>3</td><td>1/18</td><td>1/18</td><td>2/18</td><td>1/18</td><td>5/18</td></tr>
<tr><td>g(x)</td><td>1/18</td><td>6/18</td><td>4/18</td><td>4/18</td><td>1.00</td></tr>
</table>
Find
I. E(3X-2Y)
II. E(XY)
III. E(X-3)
IV. E(Y+1)</td><td rowspan="3">K3</td><td rowspan="3">CO3</td></tr>
<tr><td colspan="2">(OR)</td></tr>
<tr><td>12.b.</td><td>The joint probability density function of two dimensional random variable X and Y is given below:
$f(x,y) = 2; 0 < x < y, 0 < y < 1$,
Find cov(XY).</td></tr>
<tr><td rowspan="3">3</td><td>13.a.</td><td>The rth raw moment (moment about zero) of a random variable X. $\mu_r' = (r+1)!\,2^r$. Find cumulants of the variable X and comment on the shape characteristics of the distribution of X.</td><td rowspan="3">K4</td><td rowspan="3">CO4</td></tr>
<tr><td colspan="2">(OR)</td></tr>
<tr><td>13.b.</td><td>The joint probability density function of two dimensional random variable X_1 and X_2 is given by
$f(x_1, x_2) = e^{-(x_1+x_2)}; x_1 \geq 0, x_2 \geq 0$.
Find the characteristic function of the random variables and hence moments.</td></tr>
<tr><td rowspan="3">4</td><td>14.a.</td><td>Derive the first three moments of the Poisson distribution.</td><td rowspan="3">K4</td><td rowspan="3">CO4</td></tr>
<tr><td colspan="2">(OR)</td></tr>
<tr><td>14.b.</td><td>Write down any two properties of rectangular distribution.</td></tr>
<tr><td rowspan="3">5</td><td>15.a.</td><td>Show that $F_{n1,n2} = 1/F_{n2,n1}$.</td><td rowspan="3">K5</td><td rowspan="3">CO5</td></tr>
<tr><td colspan="2">(OR)</td></tr>
<tr><td>15.b.</td><td>Prove that normal distribution is a particular case of χ^2-distribution with $n-1$ degrees of freedom.</td></tr>
</table>

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	The two dimensional random variable X and Y have the joint probability density function $f(x,y) = \begin{cases} 21x^2y^3, 0 < x < y < 1 \\ 0, otherwise \end{cases}$ Find marginal and conditional distributions of X and Y. Also, find the probability of the event A, where $A: \{Y: y \geq \frac{1}{2}\}$.	K4	CO4
2	17	The probability distribution function of continuous random variable X is given by $f(x) = e^{-x}, x > 0$. Find moment generating function of X and hence comment on the shape of the distribution of X.	K3	CO3
3	18	The continuous random variable X has the probability distribution function $f(x) = \frac{1}{\Gamma n} e^{-x} x^{n-1}, x > 0$. Find its moments using characteristic function and comment on shape characteristic of the distribution.	K4	CO4
4	19	Derive the first four moments of the binomial distribution.	K4	CO4
5	20	Let $x_1, x_2, \dots, x_n$ be a random sample from normal population $N(\mu, \sigma^2)$. Then prove (i) $\bar{x}$ and (ii) $\frac{ns^2}{\sigma^2}$ are independently distributed as normal and chi-square respectively.	K5	CO5

Z-Z-Z END