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PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2025
(Third Semester)

Branch - **MATHEMATICS**

MAJOR ELECTIVE COURSE – II : STOCHASTIC DIFFERENTIAL EQUATIONS

Time: Three Hours | Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	Two subsets $A, B \in \mathcal{F}$ are called independent if $P(A \cap B)$ is equal to (a) P(A). P(B) (b) P(A) + P(B) (c) P(A) − P(B) (d) P(A ∪ B)	K1	CO1
	2	If $A_1, A_2, \ldots \in \mathcal{F}$ and $\{A_i\}_{i=1}^{\infty}$ is disjoint then $P(\cup_{i=1}^{\infty} A_i) =$ (a) $\sum_{i=1}^{n} P(A_i)$ (b) $\cap_{i=1}^{\infty} A_i$ (c) $\sum_{i=1}^{\infty} P(A_i)$ (d) $\cup_{i=1}^{\infty} P(A_i)$	K2	CO1
2	3	Let $f, g \in V(0,T)$ and let $0 \le S < U < T$ then $E\left[\int_S^T f dB_t\right] =$ (a) Constant (b) 0 (c) σ- Algebra (d) Measurable	K1	CO2
	4	Let $\{N_t\}_{t\ge0}$ be an increasing family of σ- Algebra of subset of Ω. A process $g(t,\omega) \times \Omega \to R^n$ is called N_t adopted if for each $t \ge 0$ the function $w \to g(t,w)$ is (a) 1-dimensional (b) F_t- measurable (c) N_t- measurable (d) B_t- measurable	K2	CO2
3	5	For each n, Let X_n be a non-negative continuous function on R such that $\int_{-\infty}^{\infty} x_n(x)dx =$ ________ (a) 0 (b) ∞ (c) −∞ (d) 1	K1	CO3
	6	The linear span of random variables of the type $xp\left\{\int_0^T h(t)\, dB_t(w) - \frac{1}{2}\int_0^T h^2(t)dt\right\}, h \in L^2(0,T)$ is (a) Continuous in $L^2(\mathcal{F}_r, P)$ (b) Dense in $L^2(\mathcal{F}_r, P)$ (c) Bounded in $L^2(\mathcal{F}_r, P)$ (d) Open in $L^2(\mathcal{F}_r, P)$	K2	CO3
4	7	$M_t = \begin{bmatrix} X_t \\ Z_t \end{bmatrix} \in R^2$ is a (a) Filtering (b) Gausssian Process (c) Martengle (d) Innovation Process	K1	CO4
	8	$E[N_t^2] =$ (a) $\int_0^t D^2(s)ds$ (b) $\int_0^\infty D^2(s)ds$ (c) $\int_0^t D(s)ds$ (d) $\int_0^t N_t ds$	K2	CO4
5	9	Let B_t be Brownian motion on R, $B_0 = 0$ and define $X_t = X_t^x = x.e^{ct+\alpha B_t}$ where c, α are constants then X_t is called (a) Brownian Process (b) Markov Process (c) Innovation Process (d) Gaussian Process	K1	CO5
	10	Let B_t^x be 1- dimensional Brownian motion starting at $x \in R^+$ put $\tau = inf\{t > 0;\ B_t^x = 0\}$ then for all $x > 0$; $E^x[\tau] =$ ________ (a) 0 (b) −∞ (c) 1 (d) ∞	K2	CO5

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SECTION - B (35 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Explain the following stochastic models : (i) Filtering problem (ii) Optimal stopping problem	K3	CO1
		(OR)		
	11.b.	If $X, Y: \Omega \to R^n$ are two given functions then prove that Y is H_x measurable if and only if there exists a Borel measurable function $g: R^n \to R^n$ such that Y= $g(X)$.		
2	12.a.	State and prove Itô isometry.	K3	CO2
		(OR)		
	12.b.	Compare the Itô Stratanovich Integrals.		
3	13.a.	State and prove 1- dimensional Itô formula.	K2	CO3
		(OR)		
	13.b.	State and prove the Itô representation theorem.		
4	14.a.	Prove that $\mathcal{L}(Z,T) = \left\{C_0 + \int_0^T f(t)dz_t\right\}; f \in L^2(0,T)C_0 \in R.$	K3	CO4
		(OR)		
	14.b.	Explain Noisy observation of a population growth.		
5	15.a.	Let $f \in C^2$ then prove that $f \in D_A$ and $Af = \sum_i b_i \frac{\partial f}{\partial x_i} + \frac{1}{2}\sum_{i,j}(\sigma\sigma^T)_{i,j}\frac{\partial^2 f}{\partial x_i \partial x_j}.$	K2	CO5
		(OR)		
	15.b.	Consider n dimensional Brownian motion $B = (B_1, \ldots, B_n)$ starting at $a = (a_1, \ldots, a_n) \in R^n (n \geq 1)$ and assume $\|a\| < R$ what is the expected value of the first exit time T_k of B from the ball $K = K_R = \{x \in R^n; \|x\| < R\}$?		

SECTION -C (30 Marks)

Answer **ANY THREE** questions
ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	State and prove Kolmogorov's extension Theorem.	K4	CO1
2	17	Let $f \in V(0,T)$ then prove that there exist a t- continuous version $\int_0^t f(s,\omega)dB_s(\omega), 0 \leq t \leq T.$	K3	CO2
3	18	Suppose $f(s,\omega)$ is continuous and of bounded variation with respect to $s \in [0,t]$ for a.a.w then prove that $\int_0^t f(s)dB_t - \int_0^t B_s df_s$.	K5	CO3
4	19	State and prove Noisy observations of a Brownian motion.	K3	CO4
5	20	State and prove the Markov property for Itô diffusions.	K4	CO5

Z-Z-Z END