22MAP312N

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2025
(Third Semester)

Branch - **MATHEMATICS**

FUNCTIONAL ANALYSIS

Time: Three Hours   Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks $(10 \times 1 = 10)$

Module No.	Question No.	Question	K Level	CO
1	1	If a normed space X contains a sequence (e_n) with $\|x-(\alpha_1 e_1+\cdots+\alpha_n e_n)\| \to 0$ as $n \to \infty$ then (e_n) is called ____. a) Infinite series b) Schander basis c) Bounded set d) Convex set	K1	CO1
	2	Let T be a bounded linear operator then the null space $N(T)$ is ______. a) bounded b) separable c) closed d) convex	K2	CO1
2	3	A Complete normed linear space is a ______. a) Banach space b) Hilbert space c) Null space d) Complete space	K1	CO2
	4	If for every $x, y \in M$, the segment joining $x \,\&\, y$ is contained in M, then the subset M of X is said to be _______. a) bounded b) separable c) convex d) closed	K2	CO2
3	5	A Partially ordered set is a set M on which there is defined a binary relation$\leq$ & satisfies a) reflexivity, antisymmetry,transitivity b) reflexivity,symmetry,transitivity c) reflexivity, symmetry d) reflexivity, transitivity	K1	CO3
	6	If $X \,\&\, Y$ be inner product spaces, $Q: X \to Y$ a bounded linear operator and $Qx = 0$ for all x then $< Qx, y >$ is equal to ______. a) 0 b) 1 c) $< Qy, y >$ d) $< Q^{*}y, x >$	K2	CO3
4	7	A vector space X is algebraically reflexive if the canonical mapping $C: X \to X^{**}$ is ______. a) injective b) bijective c) surjective d) reflexive	K1	CO4
	8	An every Hilberrt Space H is______. a) surjective b) bijective c) injective d) reflexive	K2	CO4
5	9	The ratio $d(Tx, Ty)/\, d(x, y)$ doesnot exceed a constant α which is _____. a) strictly greater than 1 b) strictly less than 1 c) greater than 1 d) less than 1	K1	CO4
	10	_______ on a metric space X is a continuous mapping. a) contraction b) fixed point c) open map d) complete space	K2	CO4

Cont...

SECTION - B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Prove that an every finite dimensional subspace Y of a normed space X is closed in X.	K2	CO1
		(OR)		
	11.b.	Prove that a finite dimensional vector space is algebraically reflexive.		
2	12.a.	State and Prove triangle inequality.	K3	CO2
		(OR)		
	12.b.	If Y is a closed subspace of a Hilbert space H then prove that $Y = Y^{\perp\perp}$.		
3	13.a.	Let $T: H_1 \to H_2$ be a bounded linear operator, where H_1 and H_2 are Hilbert spaces then prove that the Hilbert-adjoint operator T^* of T is the operator $T^*: H_2 \to H_1$ such that for all $x \in H_1 \& H_2$, $\langle Tx, y\rangle = \langle x, T^*y\rangle$ exists is unique and as a bounded linear operator with the form $\|T^*\| = \|T\|$.	K3	CO3
		(OR)		
	13.b.	Prove that an every vector space $X \neq \{0\}$ has a Hamel basis.		
4	14.a.	Prove that an every Hilbert space H is reflexive.	K4	CO4
		(OR)		
	14.b.	State and prove Closed graph theorem.		
5	15.a.	If a metric space $X = (X, d)$, where $X \neq \emptyset$. Suppose that X is a complete and let $T: X \to Y$ be a continuous on X then prove that T has precisely one fixed point.	K4	CO5
		(OR)		
	15.b.	Let $T: X \to X$ be a compact linear operator on a normal space X then prove that for every $\lambda \neq 0$ the null space $N(T_\lambda)$ of $T_\lambda = T - \lambda I$ is finite dimensional.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	If Y is a Banach space then prove that $B(X, Y)$ is a Banach space.	K4	CO1
2	17	For any subset $M \neq \varphi$ of a Hilbert space H, then prove that the span of M is dense in H if and only if $M^V = \{0\}$.	K4	CO2
3	18	State and Prove the Riesz theorem for functionals on Hilbert spaces.	K4	CO3
4	19	State and Prove Baire's Category theorem.	K4	CO4
5	20	Prove that the set of eigen values of a compact linear operator $T: X \to X$ on a normed space X is countable(perhaps finite or even empty), and the only possible point of accumulation is $\lambda = 0$.	K4	CO5

Z-Z-Z END