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PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2025
(Third Semester)

Branch - **MATHEMATICS**

DIFFERENTIAL GEOMETRY

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	The _____ at P is the line of intersection of the normal plane and the osculating plane at P. a) Tangent b) Normal c) Principal normal d) Curvature	K1	CO1
	2	The intrinsic equations of a curve express the curvature and the torsions in terms of the a) Radius b) Curve c) Tangent d) Arc length	K2	CO1
2	3	The helicoid generated by a straight line which meets the axis at right angles is a) General helicoid b) Right helicoid c) Pitch d) The anchor ring	K1	CO1
	4	One period of a _____ of pitch $2\pi a$ corresponds isometrically to a whole catenoid of parameter a) Helicoid b) Right helicoid c) Curvature d) Torson	K2	CO1
3	5	At every point of ______ its principal normal is normal to the surface. a) Helix b) Torson c) Geodesic d) Curvature	K1	CO1
	6	Geodesic curvature vector of any curve is _______ to the curve. a) Normal b) Parallel c) Orthogonal d) Coincide	K2	CO1
4	7	Second fundamental form ________ a) $Ldu^2 + 2Mdudv + Ndv^2$ b) $Edu^2 + 2Fdudv + Gdv^2$ c) $LN - M^2$ d) $\frac{L}{E} = \frac{M}{F} = \frac{N}{G}$	K1	CO1
	8	________ is Rodrigues formula a) $K = \frac{LN-M^2}{EG-F^2}$ b) $Kd\vec{r} + d\vec{N} = 0$ c) $K = K_a cos^2\varphi + K_b sin^2\varphi$ d) $(\vec{R} - \vec{r}).\vec{t} = 0$	K2	CO1
5	9	The only ______ surfaces with constant Gaussian curvature as spheres. a) complete b) compact c) ruled d) normal	K1	CO1
	10	The absolute magnitude of the total curvature of an arbitrarily large region cannot exceed ______ a) 5π b) 3π c) 2π d) 4π	K2	CO1

Cont...

SECTION - B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

pModule No.	Question No.	Question	K Level	CO
1	11.a.	Show that if a curve is given in terms of a general parameter u, then the equation of the osculating plane corresponding to $\left[\vec{R}-\vec{r}(0),\vec{r}'(0),\vec{r}''(0)\right]=0$ is $\left[\vec{R}-\vec{r},\dot{\vec{r}},\ddot{\vec{r}}\right]=0$	K3	CO2
	(OR)			
	11.b.	If the radius of the spherical curvature is constant, prove that the curve either lies on the sphere or has constant curvature.		
2	12.a.	Derive the first fundamental form.	K3	CO2
	(OR)			
	12.b.	Find the coefficients of the direction which makes an angles $\pi/2$ with the direction whose coefficients are (ℓ,m).		
3	13.a.	Prove that every helix on a cylinder is a geodesic.	K3	CO2
	(OR)			
	13.b.	Prove that if (λ,μ) is the geodesic curvature vector, then $\kappa_g=\frac{-H\lambda}{Fu'+Gv'}=\frac{H\mu}{Eu'+Fv'}$		
4	14.a.	Show that principal directions corresponding to the principal curvatures are $(EM-FL)l^2+(EN-GL)lm+(FN-GM)m^2=0$.	K3	CO3
	(OR)			
	14.b.	Prove that the edge of regression is the curve itself.		
5	15.a.	Prove that the only compact surfaces with constant Gaussian curvature are spheres.	K3	CO3
	(OR)			
	15.b.	Explain complete surfaces.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Prove that a necessary and sufficient condition for a curve to be helix is that the ratio of the curvature to torsion is constant at all points.	K4	CO4
2	17	If θ is the angle at the point *(u,v)* between the two directions given by $P\,du^2+2Q\,du\,dv+R\,dv^2=0$ then prove that $\tan\theta=\frac{2H(Q^2-PR)^{\frac{1}{2}}}{\vec{E}R-2\vec{F}Q+\vec{G}P}$	K4	CO4
3	18	Find the geodesic curvature of the parametric curve v = constant.	K4	CO4
4	19	Prove that a necessary and sufficient condition for a surface to be a developable is that its Gaussian curvature shall be zero.	K4	CO5
5	20	State and prove Hilbert's lemma.	K4	CO5

Z-Z-Z END