PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2025
(First Semester)

Branch - **MATHEMATICS**

REAL ANALYSIS

Time: Three Hours — Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	If f is differentiable in (a,b) and $f'(x)\ge 0$ for all $x\in(a,b)$, then f is ______ a) Monotonically increasing b) Constant c) Monotonically decreasing d) oscillating	K1	CO1
	2	If f is a real continuous function on [a,b] which is differentiable in (a,b), then there is a point $x\in(a,b)$ at which f(b) – f (a) = ______ a) $(b-a)f''(x)$ b) $2(b+a)f'(x)$ c) $(b-a)f'(x)$ d) $(b-a)^2 f'(x)$	K2	CO1
2	3	If P_1 *and* P_2 are two partitions, their common refinement is given by______ a) $P^* = P_1 \cap P_2$ b) $P^* = P_1 \cup P_2$ c) $P^* = P_1 - P_2$ d) $P^* = P_2 - P_1$	K1	CO2
	4	The unit step function I(x) is defined as________ a) $I(x)=1 \; for \; x>0, I(x)=2 \; for \; x\le 0$ b) $I(x)=1 \; for \; x\le 0, I(x)=-1 \; for \; x>0$ c) $I(x)=0 \; for \; x\le 0, I(x)=1 \; for \; x>0$ d) $I(x)=2 \; for \; x\le 0, I(x)=-2 \; for \; x>0$	K2	CO2
3	5	A sequence of functions $\{f_n\}$ to converge uniformly on a set E to a function f if for every $\varepsilon>0$ there is an integer N such that $n\ge N$ implies _______ for all x. a) $\lvert f_{n-2}(x)-f_{n-1}(x)\rvert \le \epsilon^2$ b) $\lvert f_{n-1}(x)-f_n(x)\rvert \le \epsilon$ c) $\lvert f_{n+1}(x)+f(x)\rvert \le \epsilon$ d) $\lvert f_n(x)-f(x)\rvert \le \epsilon$	K1	CO3
	6	If B be the uniform closure of an algebra A of bounded functions then B is a _____ a) open algebra b) uniformly closed algebra c) boundary function d) identity element	K2	CO3
4	7	$\Gamma(x) =$ _________ for 0<x<∞ a) $\int_0^\infty t^{2(x-1)}e^{-t}\,dt$ b) $\int_0^\infty t^{x-1}e^{-3t}\,dt$ c) $\int_0^\infty t^{2x-1}e^{-t}\,dt$ d) $\int_0^\infty t^{x-1}e^{-t}\,dt$	K1	CO4
	8	$E(2\pi i) =$ ________ a) 1 b) 2 c) 4 d) 6	K2	CO4
5	9	A set E of n vectors in X spans X if and only if E is __________ a) dependent b) independent c) closed d) open	K1	CO5
	10	If $A\in L(R^n,R^m)$ and $B\in L(R^m,R^k)$ then $\lVert BA\rVert \le$ ____________ a) $3\lVert B\rVert+2\lVert A\rVert$ b) $3\lVert A\rVert-2\lVert B\rVert$ c) $\lVert B\rVert\,\lVert A\rVert$ d) $\lVert B\rVert-\lVert A\rVert$	K2	CO5

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SECTION - B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Suppose f is continuous on [a,b], $f'(x)$ exists at some point x∈[a,b], g is defined on an interval I which contains the range of f, and g is differentiable at the point f(x). If h(t)=g(f(t)) (a≤t≤b),then prove that h is differentiable at x, and $h'(x)=g'(f(x))f'(x)$.	K1	CO1
		(OR)		
	11.b.	Suppose f is a real differentiable function on [a,b] and suppose $f'(a)<\lambda<f'(b)$. then prove that there is a point x∈(a,b)such that $f'(x)=\lambda$.		
2	12.a.	Prove that $f\in\Re(x)$ on [a,b] if and only if for every ∈>0 there exists a partition P such that $U(P,f,\alpha)-L(P,f,\alpha)<\varepsilon$.	K4	CO2
		(OR)		
	12.b.	Analyze the proof of fundamental theorem of calculus.		
3	13.a.	Prove that the sequence of functions $\{f_n\}$, defined on E, converges uniformly on E if and only if for every ∈> 0 there exists an integer N such that m ≥ N, n ≥ N, x ∈ E implies $\lvert f_n(x)-f_m(x)\rvert\le\varepsilon$.	K3	CO3
		(OR)		
	13.b.	Let α be monotonically increasing on [a,b]. suppose $f_n\in\Re(\alpha)$ on [a,b], for n =1,2,3,... and suppose $f_n\to f$ uniformly on [a,b] then prove that $f\in\Re(\alpha)$ on [a,b] and $\int_a^b f\,d\alpha=\lim_{n\to\infty}\int_a^b f_n\,d\alpha$.		
4	14.a.	Given a double sequence $\{a_{ij}\}$, i=1,2,3,... , j = 1,2,3,... suppose that $\sum_{j=1}^{\infty}\lvert a_{ij}\rvert=b_i\ (i=1,2,3...)$ and $\sum b_i$ converges. Then prove that $\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}a_{ij}=\sum_{j=1}^{\infty}\sum_{i=1}^{\infty}a_{ij}$.	K6	CO4
		(OR)		
	14.b.	If x > 0, and y > 0, then prove that $\int_0^1 t^{x-1}(1-t)^{y-1}dt=\frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$.		
5	15.a.	Prove that a linear operator A on a finite-dimensional vector space X is one-to-one if and only if the range of A is all of X.	K5	CO5
		(OR)		
	15.b.	If X is a complete metric space, and if ∅is a contraction of X into X, then prove that there exists one and only one x∈X such that ∅(x) = x.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Suppose f is real function on [a,b], n is a positive integer, f^{n-1} is continuous on[a,b], $f^{n}(t)$ exists for every t∈(a,b). Let α, β be distinct points of [a,b], and define $P(t)=\sum_{k=0}^{n-1}\frac{f^{(k)}(\alpha)}{k!}(t-\alpha)^k$ then prove that there exists a point x between such that $f(\beta)=P(\beta)+\frac{f^{(n)}(x)}{k!}(\beta-\alpha)^k$.	K3	CO1
2	17	If γ' is continuous on [a,b], then examine that γ is rectifiable, and $\Lambda(\gamma)=\int_a^b\lvert\gamma'(dt)\rvert$.	K4	CO2
3	18	If K is compact, if $f_n\in\varsigma(K)$ for n= 1,2,3,...., and if $\{f_n\}$is pointwise bounded and equi-continuous on K, then prove that (i) $\{f_n\}$ is uniformly bounded on K, (ii) $\{f_n\}$ contains a uniformly convergent subsequence.	K5	CO3
4	19	State and prove Parseval's theorem.	K2	CO4
5	20	Construct the proof for inverse function theorem.	K6	CO5

Z-Z-Z END