25MAP103

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2025
(First Semester)

Branch - **MATHEMATICS**

ORDINARY DIFFERENTIAL EQUATIONS

Time: Three Hours   Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions.
ALL questions carry **EQUAL** marks   (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	If b(x)=0 for all x in I the corresponding equation L(y)=0 is called a ______ a) Homogeneous equation b) Non-Homogeneous equation c) Differential Operator d) Second order equation	K1	CO1
	2	An _____ for L(y)=0 is a problem of finding a solution ϕ satisfying $\phi(x_0) = \alpha \quad \phi'(x_0) = \beta$, where x_0 is some real number, and α, β are two given constants. a) initial point b) interval value problem c) initial value problem d) integral equation	K2	CO1
2	3	If p_1, p_2, p_3, p_4 are polynomials of degree two, then they are linearly dependent on _________. a) $-\infty \leq x \leq \infty$ b) $-\infty < x <$ c) $-\infty \geq x \geq \infty$ d) $-\infty > x > \infty$	K1	CO2
	4	Let $\phi_1, \cdots, \phi_n$ be n solutions of L(y) = 0 on an interval I containing a point x_0. Then $W(\phi_1, \cdots, \phi_n)(x)$=__________ a) $e^{-a_1(x+x_0)}W(\phi_1, \cdots, \phi_n)(x)$ b) $e^{a_1(x+x_0)}W(\phi_1, \cdots, \phi_n)(x_0)$ c) $e^{-a_1(x-x_0)}W(\phi_1, \cdots, \phi_n)(x_0)$ d) $e^{-a_1(x+x_0)}W(\phi_1, \cdots, \phi_n)(x)$	K2	CO2
3	5	A linear differential equation of order n with variable coefficients is an equation of the form $a_0(x)y^{(n)} + a_1(x)y^{(n-1)} + \cdots + a_n(x)y = b(x)$, Where $a_0, a_1, \cdots a_n, b$ are complex- valued functions on some real interval I, the points where $a_0(x) = 0$ are called________ a) infinite point b) singular point c) finite point d) complex-value point	K1	CO3
	6	If the coefficients a_k of L are constants, then $W(\phi_1, \cdots, \phi_n)(x)$ =______. a) $e^{-a_1(x+x_0)}W(\phi_1, \cdots, \phi_n)(x)$ b) $e^{-a_1(x+x_0)}W(\phi_1, \cdots, \phi_n)(x_0)$ c) $e^{-a_1(x-x_0)}W(\phi_1, \cdots, \phi_n)(x_0)$ d) $e^{a_1(x+x_0)}W(\phi_1, \cdots, \phi_n)(x)$	K2	CO3
4	7	The equation $L(y) = x^k$ has a solution ψ of the form $\psi(x)$ = ______ if q(k)≠0. a) cx^k b) $cx^k \log x$ c) x^k d) $cq(k)x^k$	K1	CO4
	8	The polynomial q is called the ____for $L(y) = x^2y'' + \frac{3}{2}xy' + xy = 0$ a) polynomial b) lowest power c) complex-value d) indicial polynomial	K2	CO4
5	9	The function ϕ will be a solution of $y' = g(x)/h(x)$on I, provided_____ for all x in I a) $h(y)dy = g(y)dy$ b) $h(\phi(x)) \neq 0$ c) $h\left(\phi(x)\left(\phi'(x)\right)\right) = g(x)$ d) $h(\phi(x)) = 0$	K1	CO5
	10	If f is continuous and satisfies a ______ on R, then the successive approximations converge to a solution of the initial value problem on $\lvert x - x_0 \rvert \leq \alpha$. a) Lipschitz constant b) Euler equation c) Lipschitz condition d) Bessel function	K2	CO5

SECTION - B (35 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks   (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Let a_1, a_2 be constants, and consider the equation $L(y) = y'' + a_1y' + a_2y = 0$. If r_1, r_2 are distinct roots of the characteristic polynomial p, where $p(r) = r^2 + a_1r + a_2$, then prove that the functions φ_1, φ_2 defined by $\varphi_1(x) = e^{r_1x}$, $\varphi_2(x) = e^{r_2x}$, are solutions of $L(y) = 0$. If r_1 is repeated root of p, then the functions φ_1, φ_2 defined by $\varphi_1(x) = e^{r_1x}$, $\varphi_2(x) = xe^{r_1x}$ are solutions of $L(y) = 0$.	K2	CO1
		(OR)		

Cont...

1	11.b.	Let ϕ_1, ϕ_2 be two solutions of L(y) = 0 on an interval I, and let x_0 be any point in I. Then ϕ_1, ϕ_2 are linearly independent on I if and only if $W(\phi_1, \phi_2)(x_0) \neq 0$.	K2	CO1
2	12.a.	Let $r_1, \cdots, r_s$ be the distinct roots of the characteristic polynomial P, and suppose r_i has multiplicity m_i (thus $m_1 + m_2 + \cdots + m_s = n$). Then the n functions $e^{r_1 x}, xe^{r_1 x}, \cdots, x^{m_1 - 1}e^{r_1 x};\quad e^{r_2 x}, xe^{r_2 x}, \cdots, x^{m_2 - 1}e^{r_2 x}; \cdots;$ $e^{r_s x}, xe^{r_s x}, \cdots, x^{m_s - 1}e^{r_s x}$ are solutions of L(y) = 0.	K2	CO2
		(OR)		
	12.b.	The correspondence which associates with each $L = a_0 D^n + a_1 D^{n-1} + \cdots + a_n$ its characteristic polynomial P given by $p(r) = a_0 r^n + a_1 r^1 + \cdots + a_n$ is a one- to- one correspondence between all linear differential operators with constant coefficients and all polynomials. If L, M are associated with p, q respectively, then prove that L +M is associated with p+q, ML is associated with pq, and αL is associated with αp (α *a constant*).		
3	13.a.	Let $b_1, \cdots, b_n$ be non-negative constants such that for all x in I $\lvert a_j(x)\rvert \leqq b_j$, (j= 1, ..., n), and define k by $k = 1 + b_1 + \cdots + b_n$. If x_0 is appoint in I, and φ is a solution of L(y) = 0 on I, then prove that $\lVert\varphi(x_0)\rVert e^{-k\lvert x - x_0\rvert} \leq \lVert\varphi(x)\rVert \leq \lVert\varphi(x_0)\rVert e^{-k\lvert x - x_0\rvert}$ for all x in I.	K3	CO3
		(OR)		
	13.b.	Let b be continuous on an interval I, and let $\phi_1, \cdots, \phi_n$ be a basis for the solutions of L(y) = 0 on I. Every solution ψ of L(y) = b(x) can be written as $\psi = \psi_p + c_1\phi_1 + \cdots + c_n\phi_n$, Where ψ_p is a particular solution of L(y) = b(x), and $c_1, \cdots, c_n$ and constants. Every such ψ is a solution of L(y) = b(x). A particular solution ψ_p is given by $\psi_p(x) = \sum_{k=1}^{n} \phi_k(x) \int_{x_0}^{x} \frac{W_k(t)b(t)}{W(\phi_1, \cdots, \phi_n)(t)} dt.$		
4	14.a.	Determine a second kind for the Bessel equation of order zero.	K3	CO4
		(OR)		
	14.b.	Compute the indicial polynomials, and their roots, for the following equation: $x^2y'' + (x + x^2)y' - y = 0$		
5	15.a.	Derive Exact equations.	K3	CO5
		(OR)		
	15.b.	Suppose S is either a rectangle $\lvert x - x_0\rvert \leqq a, \quad \lvert y - y_0\rvert \leqq b, (a, b > 0)$, or a strip $\lvert x - x_0\rvert \leqq a, \quad \lvert y\rvert < \infty, \quad (a > 0)$, and that f is a real-valued function defined on S such that $\partial f/\partial y$ exists, is continuous on S, and $\left\lvert\frac{\partial f}{\partial y}(x, y)\right\rvert \leqq K, \quad ((x, y) in\ S)$, for some K>0. Then prove that f satisfies a Lipschitz condition on S with Lipschitz constant K.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	State and Prove Existence Theorem.	K3	CO1
2	17	Let ϕ be any solution of $L(y) = y^{(n)} + a_1 y^{(n-1)} + \cdots + a_n y = 0$ on an interval I containing a point x_0. Then prove that for all x in I $\lvert\lvert\phi(x_0)\rvert\rvert e^{-k\lvert x - x_0\rvert} \leq \lvert\lvert\phi(x)\rvert\rvert \leq \lvert\lvert\phi(x_0)\rvert\rvert e^{k\lvert x - x_0\rvert}$, Where $k = 1 + \lvert a_1\rvert + \cdots + \lvert a_n\rvert$.	K4	CO2
3	18	If $\phi_1, \cdots, \phi_n$ are n solutions of L(y) =0 on an interval I, they are linearly independent there if, and only if, $W(\phi_1, \cdots, \phi_n)(x) \neq 0$ for all x in I.	K3	CO3
4	19	Derive Euler equation.	K4	CO4
5	20	Let M, N be two real- valued functions which have continuous first partial derivative on some rectangle R: $\lvert x - x_0\rvert \leqq a, \quad \lvert y - y_0\rvert \leqq b$. Then prove that the equation $M(x, y) + N(x, y)y' = 0$ Is exact in R if, and only if, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ in R.	K3	CO5

Z-Z-Z END