25MAP101 / 22MAP101N

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION DECEMBER 2025
(First Semester)

Branch - **MATHEMATICS**

ALGEBRA

Time: Three Hours | Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	If $o(G) = p^2$ where p is a prime number, then G is ________. a) Cyclic b) Abelian c) Normal d) Simple	K1	CO1
	2	How many 3 –sylow sub groups can there in G if $o(G) = 72$. a) 1 b) 2 c) 3 d) 4	K2	CO1
2	3	If $p(x)$ is irreducible over F if and only if the ideal $A = (p(x))$ in $F[x]$, then A is said to be ________. a) Prime Ideal b) Left Ideal c) Right Ideal d) Maximal Ideal	K1	CO2
	4	When we say that a polynomial is said to be integer monic? a) its highest coefficient is 1 and are integers b) its highest coefficient is 2 and are integers c) its highest coefficient is 1 and are rationals d) its highest coefficient is 2 and are rationals	K2	CO2
3	5	Describe the condition that every element in W satisfies a polynomial of degree atmost mn over F, since $[W:F]$is ________. a) $< mn$ b) $\leq mn$ c) $\geq mn$ d) $> mn$	K1	CO3
	6	Give the maximum possible splitting field of $x^3 - 2$ over field of rational numbers. a) 3 b) 6 c) 9 d) 2	K2	CO3
4	7	State that any finite extension field of characteristic 0 is __________. a) Algebraic extension b) Simple extension c) Separable d) Perfect	K1	CO4
	8	If G is a group of automorphisms of K, if $a \in K$ such that $\sigma(a) = a$ for all $\sigma \in G$, then it is said to be ________ of G. a) Finite field b) Splitting field c) Extension field d) Fixed field	K2	CO4
5	9	Write the condition that the element $0 \neq v \in V$ is called a characteristic vector of T belonging to the characteristic root of $\lambda \in F$. a) $vT = I$ b) $vT = \lambda v$ c) $vT = \lambda$ d) $vT = v$	K1	CO5
	10	Interpret the rank of given real quadratic form is $x_1{}^2 + 2x_1x_2 + x_2{}^2$. a) 1 b) 2 c) 0 d) 3	K2	CO5

Cont...

SECTION - B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Show that if p is a prime number and $p/\,o(G)$, then G has an element of order p.	K2	CO1
		(OR)		
	11.b.	Show that every finite Abelian group is the direct product of cyclic groups.		
2	12.a.	State and prove the Division Algorithm of polynomials.	K3	CO2
		(OR)		
	12.b.	State and prove the Eisenstein Criterion.		
3	13.a.	(i) Explain algebraic of degree n over F (ii) Verify that if L is an algebraic extension of K and if K is an algebraic extension of F, then L is an algebraic extension of F.	K5	CO3
		(OR)		
	13.b.	Verify that a polynomial of degree n over a field can have at most n roots in any extension field.		
4	14.a.	Evaluate the following for any $f(x), g(x) \in F[x]$ and any $\alpha \in F$, 1. $(f(x)+g(x))' = f'(x)+g'(x)$. 2. $(\alpha f(x))' = \alpha f'(x)$. 3. $(f(x)\,g(x))' = f'(x)g(x) + f(x)g'(x)$	K5	CO4
		(OR)		
	14.b.	Evaluate the given polynomial $x^4+x^3+x^2+x+1$ is irreducible over rational numbers and also find that the automorphism σ of above polynomial.		
5	15.a.	Prove that if $\lambda \in F$ is a characteristic root of $T \in A(V)$, then λ is a root of the minimal polynomial of T. In particular, T only has a finite number of characteristic roots in F.	K3	CO5
		(OR)		
	15.b.	Prove that if $T \in A(V)$ has all its characteristics roots in F, then there is a basis of V in which the matrix of T is triangular.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Show that the number of p −sylow subgroups in G, for a given prime, is of the form $1+kp$.	K2	CO1
2	17	Explore that if R is a unique factorization domain and if $p(x)$ is a primitive polynomial in $R[x]$, then it can be factored in a unique ways as the product of irreducible elements in $R[x]$.	K4	CO2
3	18	Examine that if $p(x)$ is irreducible in $F[x]$ and if v is a root of $p(x)$, then $F[v]$ is isomorphic to $F'(w)$ where w is a root of $p'(t)$; moreover, this isomorphism σ can be chosen that 1. $v\,\sigma = w$, 2. $\alpha\,\sigma = \alpha'$ for every $\alpha \in F$.	K4	CO3
4	19	Evaluate that let K be the splitting field of $f(x)$ in $F[x]$ and $p(x)$ be an irreducible factor of $f(x)$ in $F[x]$. If the roots of $p(x)$ are $\alpha_1, \alpha_2, \ldots, \alpha_r$, then for each i there exists an automorphism σ_i in $G(K,F)$ such that $\sigma_i(\alpha_1) = \alpha_i$.	K5	CO4
5	20	Test the following : 1. If $T \in A(V)$ is such that $(vT, v) = 0$ for all $v \in V$, then $T = 0$. 2. The linear transformation T on V is unitary if and only if it takes an orthonormal basis of V into an orthonormal basis of V.	K5	CO5

Z-Z-Z END