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**25STU102**

# PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

## BSc DEGREE EXAMINATION DECEMBER 2025
(First Semester)

Branch- **STATISTICS**

## PROBABILITY THEORY AND RANDOM VARIABLES

Time: Three Hours | Maximum: 75 Marks

### SECTION-A (10 Marks)
Answer **ALL** questions

**ALL** questions carry **EQUAL** marks (10 × 1 = 10)

| Module No. | Question No. | Question | K Level | CO |
|---|---|---|---|---|
| 1 | 1 | The sample space associated with tossing two coins is<br>(a) {H, T} (b) {HH, HT, TH, TT}<br>(c) {H, T, H, T} (d) {HT, TH} | K1 | CO1 |
| | 2 | The classical definition of probability is applicable only when<br>(a) Events are dependent (b) Outcomes are equally likely<br>(c) Sample space is infinite (d) Outcomes are unknown | K2 | CO1 |
| 2 | 3 | A random variable is said to be discrete if it<br>(a) Takes only integer values<br>(b) Takes countable number of values<br>(c) Takes uncountable values<br>(d) None of these | K1 | CO2 |
| | 4 | The cumulative distribution function (CDF) of X is defined as<br>(a) F(x)=P(X=x) (b) F(x)=P(X>x)<br>(c) F(x)=P(X≤x) (d) F(x)=P(X<x) | K2 | CO2 |
| 3 | 5 | For two discrete random variables X and Y, the joint probability mass function is<br>(a) P(X=x) (b) P(Y=y)<br>(c) P(X=x, Y=y) (d) P(X≤x) | K1 | CO3 |
| | 6 | The conditional variance Var(X\|Y) is<br>(a) $E[(X-E[X\|Y])^2 \| Y]$ (b) Var(X)/Var(Y)<br>(c) $E[X\|Y]^2$ (d) None of these | K2 | CO3 |
| 4 | 7 | The first cumulant of a distribution is equal to<br>(a) Mean (b) Variance<br>(c) Skewness (d) Kurtosis | K1 | CO4 |
| | 8 | The probability generating function (PGF) of a discrete random variable X is defined as<br>(a) $G_X(t) = E[t^X]$ (b) $G_X(t) = E[e^{(tX)}]$<br>(c) $G_X(t) = \log(E[e^{(tX)}])$ (d) $G_X(t) = e^{(E[Xt])}$ | K2 | CO4 |
| 5 | 9 | Bernoulli's Law of Large Numbers applies to<br>(a) Continuous random variables<br>(b) Bernoulli trials<br>(c) Normal distribution<br>(d) Exponential distribution | K1 | CO5 |
| | 10 | The Central Limit Theorem states that the sum of a large number of independent random variables tends to<br>(a) Binomial distribution (b) Poisson distribution<br>(c) Normal distribution (d) Uniform distribution | K2 | CO5 |

### SECTION - B (35 Marks)
Answer **ALL** questions

**ALL** questions carry **EQUAL** Marks (5 × 7 = 35)

| Module No. | Question No. | Question | K Level | CO |
|---|---|---|---|---|
| 1 | 11.a. | Define probability and write the axioms of probability. | K2 | CO1 |
| | | (OR) | | |
| | 11.b. | State and prove Bayes theorem. | | |

**Cont...**

| | | | | |
|---|---|---|---|---|
| 2 | 12.a. | Describe discrete and continuous random variables with example. | K3 | CO2 |
| | | (OR) | | |
| | 12.b. | A random variable X has probability function as follows:<br>Values of X : -1 0 1<br>Probability : 0.2 0.3 0.5<br>Evaluate (i) E(3X+1) (ii) $E(X^2)$ | | |
| 3 | 13.a. | Define (i) Marginal Probability Distribution<br>(ii) Conditional Probability Distribution<br>(iii) Joint probability function | K4 | CO3 |
| | | (OR) | | |
| | 13.b. | Let $f(x,y) = \begin{cases} 8xy, & 0 < x < y < 1 \\ 0, & elsewhere \end{cases}$<br>Find (i) $E(Y\|X = x)$ (ii) E(XY\|X=x) | | |
| 4 | 14.a. | The probability density function of the random variable X follows the probability law:<br>$p(x) = \frac{1}{2\theta} exp\left(-\frac{\|x-\theta\|}{\theta}\right), -\infty < x < \infty$<br>Find M.G.F of X. | K3 | CO4 |
| | | (OR) | | |
| | 14.b. | Summarize about probability generating function and cumulant generating function. | | |
| 5 | 15.a. | Explain the convergence in probability. | K2 | CO5 |
| | | (OR) | | |
| | 15.b. | Examine whether the weak law of large numbers holds for the sequence ($X_k$) of independent random variables defined as follows:<br>$P(X_k = \pm 2^k) = 2^{-(2k+1)}, P(X_k = 0) = 1 - 2^{-2k}$ | | |

**SECTION -C (30 Marks)**

Answer **ANY THREE** questions

**ALL** questions carry **EQUAL** Marks (3 × 10 = 30)

| Module No. | Question No. | Question | K Level | CO |
|---|---|---|---|---|
| 1 | 16 | State and prove multiplication theorem of probability. | K2 | CO1 |
| 2 | 17 | A random variable X has the following probability mass function.<br>X: 0, 1, 2, 3, 4, 5, 6<br>P(X=x): a, 3a, 5a, 7a, 9a, 11a, 13a<br>(i) Find the value of 'a'<br>(ii) Evaluate: (a) $P(X \geq 4)$ (b) $P(X < 5)$<br>(c) $P(3 \leq X \leq 6)$ | K3 | CO2 |
| 3 | 18 | Two random variables X and Y have the following joint probability density function:<br>$f(x,y) = \begin{cases} 2 - x - y; & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0; & otherwise \end{cases}$<br>Find (i) Marginal probability density functions of X and Y;<br>(ii) Conditional density functions<br>(iii) Var(X) and Var(Y) | K4 | CO3 |
| 4 | 19 | State and prove Tchebychev's inequality. | K3 | CO4 |
| 5 | 20 | State and prove Bernoulli's weak law of large numbers. | K2 | CO5 |

Z-Z-Z END