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PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2025
(Fifth Semester)

Branch- **PHYSICS**

MATHEMATICAL PHYSICS

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	Define the divergence of a vector function. a) Rate of change of scalar function b) Measure of vector spreading out from a point c) Curl of vector d) Gradient of vector	K1	CO1
	2	Identify the correct Kronecker delta symbol value: δ_{ij} =? a) 1 if $i \neq j$, 0 if $i = j$ b) 0 if $i \neq j$, 1 if $i = j$ c) Always 0 d) Always 1	K2	CO1
2	3	State the expression for gradient in spherical coordinates. a) $\nabla f = \frac{\partial f}{\partial r}\hat{\imath} + \frac{\frac{1}{r}\partial f}{\partial \theta}\hat{\jmath} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\hat{k}$ b) $\nabla f = \frac{\partial f}{\partial x}\hat{\imath} + \frac{\partial f}{\partial y}\hat{\jmath} + \frac{\partial f}{\partial z}\hat{k}$ c) ∇f = curl f d) ∇f = divergence f	K2	CO2
	4	Indicate the correct unit vector in cylindrical coordinates along φ. a) $\hat{\rho}$ b) $\hat{\varphi}$ c) $\hat{z}$ d) $\hat{r}$	K2	CO2
3	5	Compute the eigenvalues of the matrix $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ a) 0,1 b) 2,3 c) 1,2 d) -2,-3	K1	CO3
	6	Determine the characteristic equation of matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. a) $\lambda^2 - 5\lambda + 2 = 0$ b) $\lambda^2 - 5\lambda + 10 = 0$ c) $\lambda^2 - 4\lambda + 3 = 0$ d) $\lambda^2 - 3\lambda + 2 = 0$	K2	CO3
4	7	Calculate the Laplace transform of f(t) = 1. a) 1/s b) s c) e^t d) 0	K1	CO4
	8	Apply Fourier sine transform on f(x) = x. a) $2/k^2$ b) $k^2/2$ c) 1/k d) k	K2	CO4
5	9	Explain the Cauchy-Riemann conditions for f(z) = u + iv. (K2) a) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$ b) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ c) $\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial x}, \frac{\partial u}{\partial y} = \frac{\partial v}{\partial y}$ d) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x}, \frac{\partial u}{\partial y} = \frac{\partial v}{\partial y}$	K2	CO5
	10	Recognize the analytic function among the following: f(z) = ? a) $z^2 + 1$ b) $\|z\|^2$ c) Re(z) d) Im(z)	K1	CO5

Cont...

SECTION - B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Demonstrate how to calculate the divergence of the vector field $F = x^2i + y^2j + z^2k$ and interpret its physical significance.	K3	CO1
		(OR)		
	11.b.	Determine the summation convention for the expression $A_iB_i + C_jD_j$.		
2	12.a.	Determine the arc length and volume element in a general orthogonal curvilinear coordinate system.	K3	CO2
		(OR)		
	12.b.	Derive the expression for grad ψ and curl V in orthogonal curvilinear Coordinates.		
3	13.a.	Compare the eigenvalues of $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{bmatrix}$ with its trace and determinant. Confirm the relationship between them.	K4	CO3
		(OR)		
	13.b.	Investigate the matrix $L = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ and outline the steps to diagonalize it, if possible. Point out any obstacles if it cannot be diagonalized.		
4	14.a.	Explain the following properties of Laplace transform i) Linearity property ii) change of scale property	K4	CO4
		(OR)		
	14.b.	Examine $f(x) = x e^{-bx} (x > 0)$ and determine its Fourier sine transform using substitution $x = y / b$.		
5	15.a.	Show the derivation of Cauchy-Riemann equations clearly.	K3	CO5
		(OR)		
	15.b.	Apply Morera's theorem to test whether the function $f(z) = \mid z \mid^2$ is analytic in the complex plane.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Verify stokes theorem for the vector $A = (2x-y)\,\hat{i} - yz^2\,\hat{j} - y^2z\,\hat{k}$ over the upper surface of the sphere $x^2+y^2+z^2 = 1$	K6	CO1
2	17	Analyze and derive the expressions for grad, div and curl for curvilinear co-ordinates system.	K4	CO2
3	18	Prove the Cayley Hamilton theorem for the given matrix $A=\begin{bmatrix} -11 & -10 & 6 \\ 5 & 4 & -5 \\ -20 & -20 & 4 \end{bmatrix}$	K5	CO3
4	19	Evaluate the integral $\int_0^{\pi/2} sin^{p\theta}\, cos^{q\theta}\, d\theta$ using the formula $\frac{\Gamma(\frac{p+1}{2})\Gamma(\frac{q+1}{2})}{2\Gamma(\frac{p+q+2}{2})}$ and justify why the formula is valid for the given limits.	K5	CO4
5	20	Illustrate Cauchy's Integral Theorem with a suitable diagram.	K4	CO5

Z-Z-Z END