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25MCU101

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2025
(First Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

CALCULUS

Time: Three Hours | Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	The formula for the arc length L of a space curve defined by $r(t) = < f(t), g(t), h(t) >$ for $a \le t \le b$ is ________. a) $L = \int_a^b \|r'(t)\| dt$ b) $L = \int_a^b r(t)\, dt$ c) $L = \|r'(t)\|$ d) $L = \int_a^b \|r(t)\| dt$	K1	CO1
	2	The position of a particle is given by $r(t) = < t^2, 5t, t^2 - 20t >$. What is its velocity vector at $t = 2$? a) <4, 5, -16> b) <2, 5, 2> c) <4, 5, 4> d) <4, 5, -36>	K2	CO1
2	3	For a function $z = f(x,y)$, the partial derivative with respect to x is defined as ______. a) $\frac{\partial f}{\partial x} = \lim_{h\to 0} \frac{f(x+h,y) - f(x,y)}{h}$ b) $\frac{\partial f}{\partial x} = \lim_{h\to 0} \frac{f(x+h,y+h) - f(x,y)}{h}$ c) $\frac{\partial f}{\partial x} = \frac{df}{dx}$ d) $\frac{\partial f}{\partial x} = f(x,y) - f(x+h,y)$	K1	CO2
	4	For the function $f(x,y) = 5x^2 - 3xy + 3y^2$, calculate the value of $f_x(1,2) + f_y(1,2)$. a) 11 b) 13 c) 15 d) 17	K2	CO2
3	5	The method of Lagrange multipliers is used to find the extreme values of a function $f(x,y)$ subject to a constraint $g(x,y) = k$. The necessary condition is that at the extreme point ________. a) $\nabla f = 0$ and $\nabla g = 0$ b) $\nabla f = k\,\nabla g$ c) $\nabla f = \nabla g$ d) $\nabla f = \lambda\,\nabla g$ for some scalar λ	K1	CO3
	6	Use the method of Lagrange multipliers to set up the equations for minimizing $f(x,y) = x^2 + y^2$ subject to the constraint $x + 2y = 5$, is _______. a) $2x = \lambda,\ 2y = \lambda,\ x + 2y = 5$ b) $2x = \lambda,\ 2y = 2\lambda,\ x + 2y = 5$ c) $2x = 1,\ 2y = 2,\ x + 2y = 5$ d) $x^2 = \lambda,\ y^2 = 2\lambda,\ x + 2y = 5$	K2	CO3
4	7	When evaluating a double integral over a general region, the formula for the area of a small element dA in polar coordinates (r,θ) is _______. a) $dA = dr\, d\theta$ b) $dA = r\, dr\, d\theta$ c) $dA = r^2 dr\, d\theta$ d) $dA = r^2 \sin\theta\, dr\, d\theta$	K1	CO4
	8	Which of the following represents the double integral of $f(x,y)$ over the region bounded by $y = x^2$ and $y = 4$ as an iterated integral? a) $\int_{-2}^{2}\int_{2}^{4} f(x,y)dxdy$ b) $\int_0^4 \int_{-\sqrt{y}}^{\sqrt{y}} f(x,y)dxdy$ c) Both a and b d) $\int_0^4 \int_{x^2}^{4} f(x,y)dxdy$	K2	CO4
5	9	In cylindrical coordinates, what is the relationship between the rectangular coordinates (x,y) and the cylindrical coordinates (r,θ)? a) $x = r\cos\theta,\ y = r\sin\theta$ b) $x = \rho\sin\varphi\cos\theta,\ y = \rho\sin\varphi\sin\theta$ c) $x = r,\ y = \theta$ d) $x = \cos\theta,\ y = \sin\theta$	K1	CO5

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5	10	What is the volume element dV for a triple integral in rectangular coordinates? a) $dr\ d\theta\ dz$ b) $dx\ dy\ dz$ c) $\rho^2 sin\varphi\ d\rho\ d\varphi\ d\theta$ d) $r\ dr\ d\theta\ dz$	K2	CO5

SECTION - B (35 Marks)
Answer ALL questions
ALL questions carry EQUAL Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Find parametric equations for the curve of intersection of the paraboloid $4y = x^2 + y^2$ and the plane $y = x$,	K1	CO1
		(OR)		
	11.b.	Find the length of the arc of the circular helix with vector equation $r(t) = cos\,t\ i + sint\ t$ j+ t k from (1,0,0) to $(1,0,2\pi)$		
2	12.a.	Show that $f(x,y) = xe^{xy}$ is differential at (1,0) and find its linearization.	K3	CO2
		(OR)		
	12.b.	Show that the function $u(x,y) = e^x\, sin\, y$ is a solution of Laplace equation		
3	13.a.	Use Chain rule to find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial \theta}$ where $w = xy + yz + zx$, $x = r\cos\theta, y = r\sin\theta, z = r\theta$ when $r = 2, \theta = \pi/2$.	K1	CO3
		(OR)		
	13.b.	Find the absolute maximum and minimum values of f on the set D where $f(x,y) = x + y - xy$, D is a closed triangular region with vertices (0,0),(0,2), (4,0).		
4	14.a.	Deduce $\iint_R (x - 3y^2)dA$ where $R = \{(x,y)\|0 \le x \le 2, 1 \le y \le 2\}$.	K4	CO4
		(OR)		
	14.b.	Deduce the iterated integral by converting to polar coordinates. $\int_0^{1/2}\int_{\sqrt{3}y}^{\sqrt{1-y^2}} xy^2\, dxdy$.		
5	15.a.	Deduce $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2}(x^2 + y^2)dzdydx$ using cylindrical co-ordinates.	K4	CO5
		(OR)		
	15.b.	Deduce $\iiint_B e^{(x^2+y^2+z^2)^{3/2}}\, dV$ where B is the unit ball: $B = \{(x,y,z)\|x^2 + y^2 + z^2 \le 1\}$.		

SECTION -C (30 Marks)
Answer **ANY THREE** questions
ALL questions carry EQUAL Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Sketch the curvature of the parabola $y = x^2$ at (0,0), (1,1) and (2,4).	K3	CO1
2	17	Sketch the tangent plane to the elliptic paraboloid $z = 2x^2 + y^2$ at (1,1,3).	K3	CO2
3	18	Estimate the extreme values of the function $f(x,y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$.	K2	CO3
4	19	Find the mass and center of mass of a triangular lamina with vertices (0,0),(1,0),(0,2) if the density function is $\rho(x,y) = 1 + 3x + y$.	K1	CO4
5	20	Evaluate the integral $\iint_R e^{(x+y)/(x-y)}dA$ where R is the trapezoidal region with vertices (1,0),(2,0),(0,-2) and (0,-1).	K6	CO5

Z-Z-Z END