23MCU313N

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2025
(Third Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATION**

ADVANCED MATHEMATICAL STATISTICS - I

Time: Three Hours | Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	The events whose outcomes have the same probability of occurrence are ____ events. a) Mutually exclusive b) Equally likely c) Dependent d) Complementary	K1	CO1
	2	In throwing two dice the probability of getting two numbers whose product is even is a) ½ b) ¾ c) ¼ d) 0	K2	CO1
2	3	If X can take only integer values (0,1,2,…), then X is a ___________ random variable. a) continuous b) discrete c) constant d) mixed	K1	CO2
	4	The probability of an event occurring given that another event has already occurred is a) conditional probability b) marginal probability c) joint probability d) independent probability	K2	CO2
3	5	Expectation of a constant 'C' is a) C b) 1 c) 0 d) Cx	K1	CO2
	6	If X is a random variable then E[x-E(x)] is a) E(x) b) V(x) c) E(x)-x d) 0	K2	CO3
4	7	The moment generating function is affected by change of a) origin b) scale c) both (a) and (b) d) neither (a) nor (b)	K1	CO3
	8	The characteristic function $\phi_x(t)$ is given by a) $E(e^{tx})$ b) $E\left(e^{itx}\right)$ c) $E(tx)$ d) $E(itx)$	K2	CO3
5	9	If the mean and variance of binomial distribution is 12 and 4 then the probability of success is a) 2/3 b) 4/3 c) 1/3 d) 3/4	K1	CO4
	10	The odd order moments of normal distribution are a) μ b) σ c) 0 d) 1	K2	CO4

SECTION - B (35 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	List and prove the multiplication theorem on probability, and illustrate its application with a suitable example.	K4	CO1
		(OR)		
	11.b.	Three machines A, B, and C produce 30%, 45%, and 25% of the total items, with defect rates of 1%, 2%, and 3% respectively. If a randomly selected item is found defective, find the probabilities that it was produced by machines A, B, and C.		

Cont...

<table>
<tr><td rowspan="3">2</td><td>12.a.</td><td>Define distribution function. Explain any two properties of distribution function.</td><td rowspan="3">K2</td><td rowspan="3">CO2</td></tr>
<tr><td colspan="2">(OR)</td></tr>
<tr><td>12.b.</td><td>The following table gives the joint distribution function of two discrete random variables X and Y.
<table><tr><td>X \ Y</td><td>2</td><td>4</td><td>5</td></tr><tr><td>1</td><td>1/12</td><td>1/24</td><td>1/24</td></tr><tr><td>2</td><td>1/6</td><td>1/12</td><td>1/8</td></tr><tr><td>3</td><td>1/4</td><td>1/8</td><td>1/12</td></tr></table>Find (i) P(X≤2,Y≤4); P(Y≤5) (ii) Marginal probabilities of X and Y (iii) Conditional probability of Y/X=1.</td></tr>
<tr><td rowspan="3">3</td><td>13.a.</td><td>The random variable X has the following probability distribution where a and b are some constants:
<table><tr><td>X:</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>P(X=x)</td><td>0.2</td><td>a</td><td>a</td><td>0.2</td><td>b</td></tr></table>If the E(x) =3, then find the values a and b.</td><td rowspan="3">K3</td><td rowspan="3">CO3</td></tr>
<tr><td colspan="2">(OR)</td></tr>
<tr><td>13.b.</td><td>Let X be a continuous random variable with pdf $f(x) = k(1 - x^2), -1 \leq x \leq 1$
Find k, E(X), and V(X).</td></tr>
<tr><td rowspan="3">4</td><td>14.a.</td><td>Distinguish between raw moments and central moments. Also state the limitations of mgf</td><td rowspan="3">K3</td><td rowspan="3">CO3</td></tr>
<tr><td colspan="2">(OR)</td></tr>
<tr><td>14.b.</td><td>Solve the following pdf and find the mgf, mean and variance.
$f(x) = \begin{cases} x, 0 \leq x \leq 1 \\ 2 - x, 1 \leq x \leq 2 \\ 0, elsewhere \end{cases}$</td></tr>
<tr><td rowspan="3">5</td><td>15.a.</td><td>Derive the mean and variance of Binomial distribution.</td><td rowspan="3">K4</td><td rowspan="3">CO4</td></tr>
<tr><td colspan="2">(OR)</td></tr>
<tr><td>15.b.</td><td>The number of cars passing through a tollgate per minute follows a Poisson distribution with a mean of 4. Analyse and find the probability that (i) exactly 6 cars and.(ii) less than 3 cars pass in a minute.</td></tr>
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SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	State and prove the extension of addition theorem on probability for n events	K2	CO1
2	17	Prove that the geometric mean G of the distribution f(x) = 6(2-x)(x-1), 1≤x≤2 given by 6log(16G)=19.	K3	CO2
3	18	Prove that V(X) = E[V(X|Y)] + V[E(X|Y)]	K3	CO3
4	19	Let the random variable X assume the value r with the probability law P(X=r) = qr-1p, x=0,1,2,3..... Analyse and find the moment generating function of X and hence find its mean and variance.	K4	CO3
5	20	In a normal distribution, 25% of the items are below 60, and 10% are above 85. Analyse and find the mean and standard deviation of the distribution.	K4	CO4

Z-Z-Z END