

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2025
(Fourth Semester)

Branch - MATHEMATICS WITH COMPUTER APPLICATIONS

ADVANCED MATHEMATICAL STATISTICS - II

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	Under proportional allocation of stratified random sampling, the size of the sample from each stratum depends on a) total sample size b) population size c) size of the stratum d) all of these	K1	CO1
	2	If an estimator T_n of population parameter θ as n tends to infinity is said to be: a) unbiased b) consistent c) efficient d) sufficient	K2	CO1
2	3	If a random sample X_1, X_2, \dots, X_n is drawn from a population $N(\mu, \sigma^2)$, the MLE of μ is a) θ b) \bar{x} c) σ d) σ^2	K1	CO2
	4	Generally the estimators obtained by the method of moments as compared to ML estimators are a) less efficient b) more efficient c) equally efficient d) more biased	K2	CO2
3	5	Test of hypothesis $H_0: \mu = 70$ vs $H_1: \mu > 70$ leads to a) one sided left tailed test b) two tailed test c) one sided right tailed test d) all of these	K1	CO3
	6	The size of the test is equal to the size of the a) critical region b) acceptance region c) power function d) type II error	K2	CO3
4	7	Which of the following is tested using normal distribution? a) $H_0: P = 0$ b) $H_0: P_1 = P_2$ c) $H_0: \sigma_1^2 = \sigma_2^2$ d) $H_0: O_i = E_i$	K1	CO4
	8	The test statistic used to test the equality of two population variances is a) t b) F c) χ^2 d) Z	K2	CO4
5	9	In χ^2 - goodness of fit the actual frequencies are also called as frequencies. a) theoretical b) expected c) observed d) calculated	K1	CO4
	10	Degrees of freedom for a chi-square statistic in case of contingency table of order (3X4) is a) 12 b) 6 c) 7 d) 2,3	K2	CO4

SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Summarize the different methods of non-probability sampling with examples.	K2	CO1
		(OR)		
	11.b.	State and prove the Rao-Blackwell theorem.		Cont...

2	12.a.	If X is a Poisson variate with parameter λ on the basis of the sample size n (i) find the maximum likelihood estimator, (ii) find its variance (iii) Also show that the sample mean \bar{x} is sufficient for estimating the parameter λ of the Poisson distribution.	K3	CO2															
		(OR)																	
	12.b.	Illustrate the properties of the estimates obtained by the method of moments.																	
3	13.a.	Explain the steps in the general procedure of hypothesis testing.	K3	CO3															
	13.b.	Given the frequency function $f(x, \theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}}$, $0 \leq x \leq \theta$ and that you are testing null hypothesis $H_0: \theta = 1$ and $H_1: \theta = 2$ by means of a single observed value of x . What would be the sizes of Type I and Type II errors if you choose the interval i) $0.5 \leq x \leq 2$ has critical regions? Solve and obtain power function of the test.																	
4	14.a.	Ten specimens of copper wire drawn from two large lots have the following breaking strength (in 100 kg.wt.): 5.78, 5.72, 5.70, 5.68, 5.12, 5.78, 5.70, 5.72, 5.69, 5.48. Analyze and test whether the mean breaking strength of the lot may be taken to be 5.78 (in 100 kg.wt.)	K4	CO4															
		(OR)																	
	14.b.	Examine and explain the procedure for testing the equality of two dependent samples.																	
5	15.a.	The following information is obtained concerning an investigation of 50 ordinary shops of small sizes:	K4	CO4															
		<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th rowspan="2">Shops run by</th> <th colspan="3">Shops located in</th> </tr> <tr> <th>Urban area</th> <th>Rural area</th> <th>Total</th> </tr> </thead> <tbody> <tr> <td>Men</td> <td>17</td> <td>18</td> <td>35</td> </tr> <tr> <td>Women</td> <td>3</td> <td>12</td> <td>15</td> </tr> <tr> <td>Total</td> <td>20</td> <td>30</td> <td>50</td> </tr> </tbody> </table> <p>Can it be concluded that there is no significant association between the location of the shops and the gender of the shop owners?</p>			Shops run by	Shops located in			Urban area	Rural area	Total	Men	17	18	35	Women	3	12	15
Shops run by	Shops located in																		
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	15.b.	Analyze the procedure for testing the independence of attributes in a 4×3 contingency table.																	

SECTION -C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks $(3 \times 10 = 30)$

Module No.	Question No.	Question	K Level	CO																		
1	16	State and prove the Cramer-Rao inequality	K4	CO1																		
2	17	In a random sample from normal distribution $N(\mu, \sigma^2)$, find the maximum likelihood estimators for i) μ when σ^2 is known, ii) σ^2 when μ is known, iii) Simultaneous estimation of μ and σ^2 .	K4	CO2																		
3	18	Examine and derive the probability function of Snedecor's F-distribution, including the necessary steps and assumptions.	K4	CO3																		
4	19	Below are given the gain in weight (lb.) of patients on two diets x and y. Gain in weight (lb.)	K4	CO4																		
		<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td>X</td><td>8</td><td>12</td><td>13</td><td>9</td><td>3</td><td></td><td></td> </tr> <tr> <td>Y</td><td>10</td><td>8</td><td>12</td><td>15</td><td>6</td><td>8</td><td>11</td> </tr> </table> <p>Can it be concluded at 5% level whether the two diets differ significantly with regards to increase in the mean weight.</p>			X	8	12	13	9	3			Y	10	8	12	15	6	8	11		
X	8	12	13	9	3																	
Y	10	8	12	15	6	8	11															
5	20	300 digits were chosen at random and found to give the following distribution	K4	CO4																		
		<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td>Digits</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td> </tr> <tr> <td>Frequency</td><td>18</td><td>32</td><td>28</td><td>34</td><td>42</td><td>50</td><td>17</td><td>23</td><td>27</td><td>29</td> </tr> </table> <p>Test the hypothesis that the digits were distributed as equal numbers in the table from which the data were collected.</p>			Digits	0	1	2	3	4	5	6	7	8	9	Frequency	18	32	28	34	42	50
Digits	0	1	2	3	4	5	6	7	8	9												
Frequency	18	32	28	34	42	50	17	23	27	29												