24MCU521/23MCU520

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2025
(Fifth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

REAL ANALYSIS

Time: Three Hours　　　　Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks　　$(10 \times 1 = 10)$

Module No.	Question No.	Question	K Level	CO
1	1	A real valued function is said to be strictly increasing if for any x,y in the domain of f, x<y, then_______. a) f(x)<f(y)　b) f(x)>f(y)　c) f(x)=f(y)　d) f(x)>0	K1	CO1
	2	$lt_{x\to 1}\sqrt{x+3} =$ ________. a) 2　b) 1　c) 4　d) 3	K2	CO1
2	3	A function is continuous at $a \in R$ if $\lim_{x\to a} f(x) =$ ______. a) f(a)　b) ∞　c) 0　d) f(-a)	K1	CO2
	4	The set of all irrationals is of _________ category. a) first category　b) second category c) n^{th} category　d) third category	K2	CO2
3	5	If f is a continuous real valued function on [a,b] then f takes every value between _____ and ______. a) f(a) & f(b)　b) -∞ & ∞　c) 0 & ∞　d) -∞ & 0	K1	CO3
	6	Every bounded subset of R^2 is _______. a) totally bounded　b) compact c) bounded　d) unbounded	K2	CO3
4	7	If the metric M has the Heine Borel property then M is____. a) compact　b) bounded c) complete　d) unbounded	K1	CO4
	8	Every finite subset of any metric space is ____. a) bounded　b) totally bounded c) unbounded　d) compact	K2	CO4
5	9	A bounded function has Riemann Integral if f is ________ atmost every point. a) continuous　b) differentiable c) both a &b　d) none of the above	K1	CO5
	10	If a < b, then (a,b) is not of measure ________. a)0　b) 1　c) -1　d)2	K2	CO5

SECTION - B (35 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** Marks　　$(5 \times 7 = 35)$

Module No.	Question No.	Question	K Level	CO
1	11.a.	Prove that $\lim_{x\to a}[f(x) + g(x)] = L + M$, if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a}[g(x)] = M$.	K3	CO1
		(OR)		
	11.b.	If f is a monotonic function on (a,b) and $c \in (a,b)$ then $\lim_{x\to c+} f(x)$ and $\lim_{x\to c-} f(x)$ both exist.		

Cont…

2	12.a.	Prove that $g \circ f$ is continuous at 'a' if f, g are real valued and f is continuous at a and g is continuous at f(a).	K3	CO2
		(OR)		
	12.b.	Prove that the set of all irrationals is not of type F_a.		
3	13.a.	If f is a continuous function from a metric space M_1 into M_2 and if M_1 is connected prove that the range of f is also connected.	K3	CO3
		(OR)		
	13.b.	Prove that, if the subset A of the metric space $< M, \rho >$ is totally bounded then A is bounded.		
4	14.a.	If A is a closed subset of the compact metric $< M, \rho >$ then prove that the metric space $< A, \rho >$ is also compact.	K3	CO4
		(OR)		
	14.b.	Prove that the range $f(M_1)$ of f is compact, where f is a continuous function from the compact metric space M_1 into M_2.		
5	15.a.	State and prove Rolle's theorem.	K3	CO5
		(OR)		
	15.b.	Prove that if f $\in$([a,b]), then $\lvert f \rvert \in R[a,b]$ and $\left\lvert \int_a^b f \right\rvert \leq \int_a^b \lvert f \rvert$.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks $(3 \times 10 = 30)$

Module No.	Question No.	Question	K Level	CO
1	16	Let $< M, \rho >$ be a metric space and if $\{s_n\}_{n=1}^{n=\infty}$ is a convergent sequence of points of M then, prove that $\{s_n\}_{n=1}^{n=\infty}$ is Cauchy.	K3	CO1
2	17	Prove that $A \cup B$ is of the first category if A and B are sets of the first category.	K3	CO2
3	18	Prove that $A \subset M$ is totally bounded if and only if every sequence of points of A contains a Cauchy subsequence *where* $< M, \rho >$ is a metric space.	K3	CO3
4	19	Prove that metric space M is compact if m has the Heine Borel property.	K3	CO4
5	20	Prove that f is continuous at c if the real valued function f has a derivative at $c \in R^1$.	K3	CO5

Z-Z-Z END