

**PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)**

**BSc DEGREE EXAMINATION DECEMBER 2025
(Third Semester)**

**Branch – MATHEMATICS WITH COMPUTER APPLICATIONS
PARTIAL DIFFERENTIAL EQUATIONS AND FOURIER SERIES**

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

Question No.	Question	K Level	CO
1	Find the Partial Differential Equations by eliminating a and b from the equation $z = ax + by + a^2 + b^2$ is _____ a) $z = x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$ b) $z = x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$ c) $z = x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial x} + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$ d) $z = x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$	K1	CO1
2	Find the solution of $p + 3q = 5z + (y - 3x) =$ _____ a) $5x - \log[5z + \tan(y - 3x)] = \Phi(y - 3y)$ b) $5x - \log[5z + \tan(y - 3z)] = \Phi(y - 3y)$ c) $5x - \log[5z + \tan(y - 3x)] = \Phi(y - 3x)$ d) $5x - \log[5x + \tan(y - 3x)] = \Phi(y - 3y)$	K2	CO1
3	Find the complete integral of $q = 3p^2$. a) $z = ay + 3a^2y + b$ b) $z = az + 3a^2y + b$ c) $z = ax + 3a^2z + b$ d) $z = ax + 3a^2y + b$	K1	CO2
4	Find the complete integral of $q = px + p^2$. a) $z = \frac{x^2}{4} \pm \frac{1}{2} \left[\frac{x}{2} \sqrt{(x^2 + 4a)} + 2 \log \left\{ x + \sqrt{(x^2 + 4a)} \right\} \right] + ay + b$ b) $z = \frac{x^2}{4} \pm \frac{1}{2} \left[\frac{x}{2} \sqrt{(x^2 + 5a)} + 2 \log \left\{ x + \sqrt{(x^2 + 4a)} \right\} \right] + ay + b$ c) $z = \frac{x^2}{4} \pm \frac{1}{2} \left[\frac{x}{2} \sqrt{(x^2 + 4a)} + 2 \log \left\{ x + \sqrt{(x^2 + 5a)} \right\} \right] + ay + b$ d) $z = \frac{x^2}{4} \pm \frac{1}{2} \left[\frac{x}{2} \sqrt{(x^2 + 4a)} + \log \left\{ x + \sqrt{(x^2 + 4a)} \right\} \right] + ay + b$	K2	CO2
5	Find the solution of $r = \sin(xy)$. a) $z = -\left(\frac{1}{y^2}\right) \sin(xy) + x\Phi_1(y) + \Phi_2(y)$ b) $z = -\left(\frac{1}{y^2}\right) \sin(x) + x\Phi_1(y) + \Phi_2(y)$ c) $z = -\left(\frac{1}{y^2}\right) \sin(y) + x\Phi_1(y) + \Phi_2(y)$ d) $z = -\left(\frac{1}{y^2}\right) \sin(xy) + x\Phi_1(x) + \Phi_2(x)$	K1	CO3
6	Find the solution of $t - 2xq + x^2z = (x - 2)e^{3x+2y}$. a) $z = e^{xy} \{ \Phi_1(y) + x\Phi_2(y) \} + \frac{e^{3x+2y}}{x-2}$ b) $z = e^{xy} \{ \Phi_1(x) + x\Phi_2(x) \} + \frac{x-2}{e^{3x+2y}}$ c) $z = e^{xy} \{ \Phi_1(x) + x\Phi_2(x) \} + \frac{x-2}{e^{3x+2y}}$ d) $z = e^x \{ \Phi_1(x) + x\Phi_2(x) \} + \frac{x+2}{e^{3x+2y}}$	K2	CO3
7	The value of $\cos 2\pi =$ _____. a) 0 b) 2 c) 1 d) -1	K1	CO4
8	If $f(x)$ is an even function then $f(-x) =$ _____. a) $f(-x)$ b) $f(0)$ c) $f(1)$ d) $f(x)$	K2	CO4
9	$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ is called _____. a) Two dimensional wave equation b) One dimensional heat equation c) Two dimensional heat equation d) One dimensional wave equation	K1	CO5
10	$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ is called _____. a) One dimensional heat equation b) Two dimensional wave equation c) Two dimensional heat equation d) One dimensional wave equation	K2	CO5

Cont...

SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks (5 × 7 = 35)

Question No.	Question	K Level	CO
11.a.	Eliminate the arbitrary constants from $z = (x - a)^2 + (y - b)^2$ to form the partial differential equation.	K3	CO1
	(OR)		
11.b.	Solve $z(z^2 + xy)(px - qy) = x^4$.		
12.a.	Show that the equation $xp - yq = x$ and $x^2p + q = xz$ are compatible and find their solution.	K2	CO2
	(OR)		
12.b.	Find complete integral of $p^2 + q^2 - 2pq \tanh 2y = \operatorname{sech}^2 2y$.		
13.a.	Solve $ys + p = \cos(x + y) - y \sin(x + y)$.	K3	CO3
	(OR)		
13.b.	Solve $s - t = \frac{x}{y^2}$.		
14.a.	State and prove orthogonality of the trigonometric system.	K3	CO4
	(OR)		
14.b.	Find the fourier series of the function $f(x) = \begin{cases} 0 & \text{if } -2 < x < -1 \\ k & \text{if } -1 < x < 1 \\ 0 & \text{if } 1 < x < 2 \end{cases}$ $p = 2L = 4, L = 2$		
15.a.	Discuss the steps for finding solutions by separating variables using of fourier series.	K4	CO5
	(OR)		
15.b.	Find the temperature $u(x, t)$ in a laterally insulated copper bar 80cm long if the initial temperature is $100 \sin\left(\frac{\pi x}{80}\right)^\circ\text{C}$ and the ends are kept at 0°C . How long will it take for the maximum temperature in the bar to drop to 50°C ? First guess, then calculate. Physical data for copper: density 8.92g/cm^3 , specific heat $0.092\text{cal/(g}^\circ\text{C)}$, thermal conductivity $0.95\text{ cal/(cm sec}^\circ\text{C)}$.		

SECTION - C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks (3 × 10 = 30)

Question No.	Question	K Level	CO
16	Form the Partial Differential Equations by eliminating the arbitrary function ϕ from $\phi(x^2 + y^2 + z^2, z^2 - 2xy) = 0$.	K3	CO1
17	Find the complete integral of the equation $q = \left\{ \frac{1+p^2}{1+y^2} \right\} x + yp(z - px)^2$.	K3	CO2
18	Solve $xr + ys + p = 10xy^3$.	K5	CO3
19	Find the two half range expansions of the function $f(x) = \begin{cases} \frac{2k}{L}x & \text{if } 0 < x < \frac{L}{2} \\ \frac{2k}{L}(L - x) & \text{if } \frac{L}{2} < x < L \end{cases}$	K4	CO4
20	Find the temperature in a laterally insulated bar of length L whose ends are kept at temperature 0, assuming that the initial temperature is $f(x) = \begin{cases} x & \text{if } 0 < x < \frac{L}{2} \\ L - x & \text{if } \frac{L}{2} < x < L \end{cases}$ assuming that the ends are insulated (instead of being kept at temperature 0).	K4	CO5