**25MCU102/ 22MCU102N**

**PSG COLLEGE OF ARTS & SCIENCE**
(AUTONOMOUS)

**BSc DEGREE EXAMINATION DECEMBER 2025**
(First Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

# ORDINARY DIFFERENTIAL EQUATIONS AND LAPLACE TRANSFORMS

Time: Three Hours  Maximum: 75 Marks

## SECTION-A (10 Marks)

Answer ALL questions
ALL questions carry **EQUAL** marks  (10 × 1 = 10)

| Module No. | Question No. | Question | K Level | CO |
|---|---|---|---|---|
| 1 | 1 | The general solution $y$ of the differential equation $\frac{dy}{dx} = k$, is_____.<br>a) $y = Ce^{kt}$<br>b) $y = Ce^{-kt}$<br>c) $y = kCe^{t}$<br>d) $y = kt + C$ | K2 | CO1 |
| | 2 | For Initial Value Problem $\frac{dy}{dx} = x - y, y(0) = 1$, using Euler's method with step size $h = 0.1, y(0.1)$ is _______.<br>a) 0.90  b) 0.91  c) 0.99  d) 1.10 | K2 | CO1 |
| 2 | 3 | If the roots $r_1, r_2, \dots, r_n$ of the characteristic equation $a_n r^n + a_{n-1}r^{n-1} + \cdots + a_1 r + a_0 = 0$ are real and distinct then the general solution of $a_n y^n + a_{n-1}y^{n-1} + \cdots + a_1 y + a_0 = 0$ is ________<br>a) $y(x) = c_1 e^{r_1 x} + c_2 e^{r_2 x} + \cdots + c_n e^{r_n x}$<br>b) $y(x) = c_1 e^{r_1 x} - c_2 e^{r_2 x} - \cdots - c_n e^{r_n x}$<br>c) $y(x) = e^{r_1 x} + e^{r_2 x} + \cdots + e^{r_n x}$<br>d) None | K2 | CO2 |
| | 4 | The existence and uniqueness theorem for $\frac{dy}{dx} = f(x,y), y(x_0) = y_0$ guarantees a unique solution provided __________.<br>a) F is continuous at $(x_0, y_0)$<br>b) $\frac{\partial f}{\partial x}$ is continuous at $(x_0, y_0)$<br>c) $\frac{\partial f}{\partial y}$ is continuous at $(x_0, y_0)$<br>d) Both $f$ and $\frac{\partial f}{\partial y}$ are continuous at $x_0, y_0$ | K2 | CO2 |
| 3 | 5 | Which of the following differential equations is exact?<br>a) $(y + x)dx + (x - y)dy = 0$<br>b) $(2xy + y^2)dx + (x^2 + 2x\,y)dy = 0$<br>c) $(x^2 + y)dx + (x2 + y)dy = 0$<br>d) $(y^2 + x)dx + (2xy)dy = 0$ | K2 | CO3 |
| | 6 | Which of the following linear equations forms the Bernoulli equation $\frac{dy}{dx} + \frac{1}{x}y = y^2$ under the substitution $u = y^{-1}$?<br>a) $\frac{du}{dx} + \frac{1}{x}u = 1$  b) $\frac{du}{dx} - xu = -1$<br>c) $\frac{du}{dx} - \frac{1}{x}u = 1$  d) $\frac{du}{dx} + \frac{1}{x}u = -1$ | K2 | CO3 |
| 4 | 7 | The general solution of $y'' - 4y + 4y = 0$ is<br>a) $y = C_1 e^{2x} + C_2 e^{2x}$  b) $y = C_1 + C_2 x e^{-2x}$<br>c) $y = (C_1 + C_2 x)e^{2x}$  d) None | K2 | CO4 |
| | 8 | For the mass-spring-damper equation $y'' + 2y' + 5y = 0$, the system behaviour is<br>a) Overdamped  b) Critically damped<br>c) Unstable  d) underdamped | K2 | CO4 |

**Cont...**

| | | | | |
|---|---|---|---|---|
| 5 | 9 | In using undetermined coefficients to solve $y'' - 2y + y = e^x$ the correct form of a particular solution $y_p$ is ________.<br>a) $y_p = A\,xe^x$ b) $y_p = A\,x^2e^x$<br>c) $y_p = A\,x^2$ d) None | K2 | CO5 |
| | 10 | Which method is appropriate for finding a particular solution of $y'' + y = \sec x$?<br>a) Undetermined coefficients b) Separation of variables<br>c) Variation of parameters d) Exact equations | K2 | CO5 |

**SECTION – B (35 Marks)**

Answer **ALL** questions

**ALL** questions carry **EQUAL** Marks (5 × 7 = 35)

| Module No. | Question No. | Question | K Level | CO |
|---|---|---|---|---|
| 1 | 11.a. | Solve the initial value problem $\frac{dy}{dx} = 2x + 3,\ y(1) = 2$. | K3 | CO1 |
| | | (OR) | | |
| | 11.b. | Solve $\frac{dy}{dx} = \sqrt{x},\ y(4) = 0$. | | |
| 2 | 12.a. | Derive the principle of Superposition for homogeneous equation. | K3 | CO2 |
| | | (OR) | | |
| | 12.b. | Show that the functions $y_1(x) = e^{-3x},\ y_2(x) = \cos 2x$ and $y_3(x) = \sin 2x$ are linearly independent. | | |
| 3 | 13.a. | Find the general solution of fifth order differential equation, $9y^{(5)} - 6y^{(4)} + y^{(3)} = 0$. | K3 | CO3 |
| | | (OR) | | |
| | 13.b. | Suppose the RLC circuit with $I(0) = Q(0) = 0$, is connected at time $t = 0$ to a battery supplying a constant 110V then find the current in the circuit. | | |
| 4 | 14.a. | Derive the linearity of the Laplace Transform. | K2 | CO4 |
| | | (OR) | | |
| | 14.b. | Compute $\mathcal{L}\{t \sin kt\}$. | | |
| 5 | 15.a. | Express the convolution of $\cos t$ and $\sin t$ i.e., $(\cos t) * (\sin t)$ | K2 | CO5 |
| | | (OR) | | |
| | 15.b. | Compute $\mathcal{L}^{-1}\left\{\tan^{-1}\left(\frac{1}{s}\right)\right\}$. | | |

**SECTION -C (30 Marks)**

Answer **ANY THREE** questions

**ALL** questions carry **EQUAL** Marks (3 × 10 = 30)

| Module No. | Question No. | Question | K Level | CO |
|---|---|---|---|---|
| 1 | 16 | Show that the general solution of $(x^2 + 1)\frac{dy}{dx} + 3xy = 6x$. | K3 | CO1 |
| 2 | 17 | Elaborate and derive the Wronskians of solutions. | K3 | CO2 |
| 3 | 18 | Find the particular solution of $y'' + y = \tan x$. | K2 | CO3 |
| 4 | 19 | Consider a mass-and-spring system with $m = \frac{1}{2},\ k = 17$, and $c = 3$ in mks units. As usual, let $x(t)$ denote the displacement of the mass $m$ from its equilibrium position. If the mass is set in motion with $x(0) = 3$ and $x'(0) = 1$, find $x(t)$ for the resulting damped free oscillations. | K2 | CO4 |
| 5 | 20 | Find $\mathcal{L}\{(\sinh t)/t\}$. | K3 | CO5 |

Z-Z-Z END