23MCU312N

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2025
(Third Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATION**

DISCRETE MATHEMATICS AND GRAPH THEORY

Time: Three Hours | Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	Choose the Absorption Law from the following data: a) $p\vee(q\wedge p) \Leftrightarrow p$ b) $p\vee(p\wedge q) \Leftrightarrow p$ c) $p\vee(q\wedge p) \Leftrightarrow q$ d) $p\vee(p\wedge q) \Leftrightarrow q$	K1	CO1
	2	Classify the Universally quantified statement $\forall x[p(x) \rightarrow q(x)]$ to be $\forall x[q(x) \rightarrow p(x)]$ is said to be __________. a) Contrapositive b) Inverse c) Converse d) Universe	K2	CO1
2	3	Find the coefficient of x^4 in the generating function $(x) = \sum_{i=1}^{\infty}[\frac{x}{1-x}]^i$. a) 8 b) 7 c) 6 d) 5	K1	CO2
	4	Classify the function e^x is the __________ for the sequence 1,1,1, ... a) Generating function b) Exponential function c) Ordinary generating function d) Exponential generating function	K2	CO2
3	5	Define the recurrence relation of the following sequence 3,7,11,15,19, ... a) $a_n = a_{n-1} + 2$ b) $a_n = a_{n-1} + 3$ c) $a_n = a_{n-1} + 4$ d) $a_n = a_{n-1} + 5$	K1	CO3
	6	Interpret the solution of the $a_n - 3a_{n-1} = 5(3^n)$, where $n \geq 1$ and $a_0 = 2$. a) $a_n = (2 + 5n)(3^n), n \geq 0$ b) $a_n = (3 + 5n)(3^n), n \geq 0$ c) $a_n = (2 + 5n)(2^n), n \geq 0$ d) $a_n = (3 + 5n)(2^n), n \geq 0$	K2	CO3
4	7	How many paths of length 4 are there in the complete graph k_7. a) 1230 b) 1240 c) 1250 d) 1260	K1	CO4
	8	An undirected graph where each vertex has the same degree is called a __________. a) Connected graph b) Regular graph c) Directed graph d) Bipartite graph	K2	CO4
5	9	What is the sum of the degrees of all the vertices in tree with 2000 vertices? a) 3997 b) 3999 c) 3998 d) 3996	K1	CO5
	10	Let T be a balanced complete m-ary tree with ℓ leaves. Illustrate the height of T is __________ . a) $log_m \ell$ b) $log_\ell m$ c) $log_n \ell$ d) $log_m n$	K2	CO5

SECTION - B (35 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Interpret the validity of the argument $p \rightarrow q$ $q \rightarrow (r\wedge s)$ $\neg r\vee(\neg t\vee u)$ $p\wedge t$ __________ $\therefore u$	K2	CO1
		(OR)		

Cont...

	11.b.	Let $g: B^4 \to B$, where $g(w,x,y,z) = (w+x+y)(x+\bar{y}+z)(w+\bar{y})$. To interpret the Conjunctive Normal Form (CNF) for g.		
2	12.a.	Simplify the generating function for the number of n- combinations of apples, bananas, oranges and pears wherein each n- combination the number of apples is even, the number of bananas is odd, the number of oranges is between 0 and 4 and there is atleast one pear.	K4	CO2
		(OR)		
	12.b.	Simplify a formula for $\sum_{k=1}^{n} k$ using the generating function for the sequence 0,1,3,6,10,15, ...		
3	13.a.	A person invests 100,000 at 12% interest compounded annually. Solve it (i) Find the amount at the end of $1st, 2nd, 3rd$ year. (ii) Write the general explicit formula. (iii) How long will it take to double the investment.	K3	CO3
		(OR)		
	13.b.	Solve the recurrence relation $a_n = 2(a_{n-1} - a_{n-2})$, where $n \geq 2$ and $a_0 = 1, a_1 = 2$.		
4	14.a.	(i) Define Digraph, Circuit and Path. (ii) Let $G = (V, E)$ be an undirected graph, with $a, b \in V, a \neq b$. If there exists a trail (in G) from a to b, there is a path (in G) from a to b.	K3	CO4
		(OR)		
	14.b.	Let $G = (V, E)$ be a connected planar graph or multigraph with $\lvert V\rvert = v$ and $\lvert E\rvert = e$. Let r be the number of regions in the plane determined by a planar embedding of G; one of these regions has infinite area is called the infinite region. Then identify that if $v - e + r = 2$.		
5	15.a.	(i) If $G = (V, E)$ is an undirected graph, then G is connected if and only if G has a spanning tree. (ii) For every tree $T = (V, E)$, if $\lvert V\rvert \geq 2$, then T has at least two pendant vertices.	K4	CO5
		(OR)		
	15.b.	(i) Define Binary tree and Complete Binary tree. (ii) Let $T = (V, E)$ be a completed m-ary tree of height h with ℓ leaves. Then $\ell \leq m^h$ and $h \geq \lceil log_m \ell \rceil$.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Simplify the following statement by using truth table. (i) $(p \to q) \wedge [(q \wedge \neg r) \to (p \vee r)]$. (ii) $p \vee (q \wedge r)$ and $(p \vee q) \wedge r$.	K4	CO1
2	17	A ship carries 48 flags, 12 each of the colors are red, white, blue and black. Twelve of these flags are placed on a vertical pole in order to communicate a signal to other ships. Analyze (i) How many of these signals use an even number of blue flags and an odd number of black flags? (ii) How many of the signals have at least three white flags or no white flags at all?	K4	CO2
3	18	Determine the formula for the sum of the cubes of the first n natural numbers using a recurrence relation.	K4	CO3
4	19	Analyze that (i) Let $G = (V, E)$ be a loop-free undirected graph, $\lvert V\rvert = n \geq 3$. If $deg(x) + deg(y) \geq n$ for all nonadjacent $x, y \in V$, then G contains a Hamilton cycle. (ii) If $G = (V, E)$ is a loop-free undirected graph with $\lvert V\rvert = n \geq 3$, and if $\lvert E\rvert \geq \binom{n-1}{2} + 2$, then G has a Hamilton cycle.	K4	CO4
5	20	Determine that let $G = (V, E)$ be a loop-free connected undirected graph with $T = (V, E')$ a depth-first spanning tree for G. Let r be the root of T and $v \in V, v \neq r$. Then v is an articulation point of G if and only if there exists a child c of v with no back edge (relative to T in G) from a vertex in T_c, the subtree rooted c, to an ancestor of v.	K4	CO5

Z-Z-Z END