TOTAL PAGES : 2

24MCU520/23MCU519

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2025
(Fifth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

ABSTRACT ALGEBRA

Time: Three Hours — Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions

ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	A function $f : A \to B$ is called onto if a) Every element of A maps to a unique element of B b) Every element of B has a preimage in A c) Both a and b d) None	K2	CO1
	2	The greatest common divisor of 24 and 26 is a) 6 b)12 c) 18 d) 24	K2	CO1
2	3	The group of integers under addition is a) Finite group b) Infinite cyclic group c) Non-abelian group d) Symmetric group	K2	CO2
	4	If H is a normal subgroup of G, then for all $g \in G$ a) $gH = Hg$ b) $gH \neq Hg$ c) $H = \{e\}$ d) None	K2	CO2
3	5	A group homomorphism preserves a) Only the identity element b) The group operation c) Only inverses d) None	K2	CO2
	6	The kernel of a homomorphism is always a) A subgroup of domain b) A subgroup of co-domain c) Always trivial d) Equal to the whole group	K1	CO2
4	7	A commutative ring with identity and no zero divisors is called a) Field b) Integral domain c) Group d) Module	K1	CO3
	8	In any ring, $0.a =$ ______. a) 0 b) a c) 1 d)-a	K1	CO3
5	9	The degree of the product of two polynomials is a) Sum of their degrees b) Product of their degrees c) Maximum of their degrees d) Minimum if their degrees	K1	CO4
	10	Over $\mathbb{Z}$, the polynomial $x^2 + 1$ is a) Reducible b) Irreducible c) Both d) None	K1	CO4

SECTION - B (35 Marks)
Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	State and prove mapping of sets holds associative law.	K1	CO1
		(OR)		
	11.b.	Define one to one mapping and list any 4 examples.		

Cont...

2	12.a.	Show that if ϕ is a homomorphism of G into $\bar{G}$ then i) $\phi(e) = \bar{e}$, the unit element of $\bar{G}$. ii) $\phi(x^{-1}) = \phi(x)^{-1}$ for all $x \in G$.	K2	CO2
		(OR)		
	12.b.	If ϕ is a homomorphism of G into $\bar{G}$ with kernel K, then K is a normal subgroup of G.		
3	13.a.	Show that every permutation can be uniquely expressed as a product of disjoint cycles.	K3	CO3
		(OR)		
	13.b.	Write the permutations for $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 4 & 5 & 1 & 6 & 7 & 9 & 8 \end{pmatrix}$ as the product of disjoint cycles.		
4	14.a.	Prove that a finite integral domain is a field.	K3	CO3
		(OR)		
	14.b.	If p is a prime number then J_p, then prove that ring of integers $mod\ p$, is a field.		
5	15.a.	Suppose R be a Euclidean ring, then any two elements a and b in R have a greatest common divisor d, moreover $d = \lambda a + \mu b$ for some $\lambda, \mu \in R$.	K4	CO2
		(OR)		
	15.b.	Prove that R be an integral domain with unit element and suppose that for $a, b \in R$ both $a\|b$ and $b\|a$ are true, then $a = ub$ where u is a unit in R.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Show that if G is group, then i) The identity element of G is unique ii) Every $a \in G$ has a unique inverse in G iii) For every $(a^{-1})^{-1} = a$ iv) For all $a, b \in G, (ab)^{-1} = b^{-1}a^{-1}$	K2	CO1
2	17	Prove that the subgroup N of G is normal subgroup of G if and only if every left coset of N is right coset of N in G.	K4	CO2
3	18	State and Prove Cayley's Theorem.	K3	CO3
4	19	If R is a ring, then for all $a, b \in R$, prove that i) $a0 = 0a = a$ ii) $a(-b) = (-a)b = -(ab)$ iii) $(-a)(-b) = ab$	K3	CO3
5	20	Prove that $J[i]$ is an Euclidean Ring.	K4	CO4

Z-Z-Z END