22MAU515N

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2025
(Fifth Semester)

Branch – **MATHEMATICS**

LINEAR ALGEBRA

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	State the condition that the matrix is said to be a symmetric matrix. a) $A = A^2$ b) $A = A^T$ c) $-A = A^T$ d) $A^2 = 1$	K1	CO1
	2	Interpret the determinant value of the given matrix $A = \begin{bmatrix} 2 & 2 \\ 3 & 5 \end{bmatrix}$. a) 7 b) 8 c) 9 d) 4	K2	CO1
2	3	If T is an homomorphism of a vector space U to V and T is one to one then T is ________. a) Isomorphism b) Homeomorphism c) Isomorphic d) Automorphism	K1	CO2
	4	Given that if S is a subset of a vector space V and if S consists of linearly independent elements and $V = L(S)$ then S said to be ________ of V. a) Subspace b) Vectorspace c) Linear span d) Basis	K2	CO2
3	5	Enumerate the $dim_F Hom(V, V)$ if $dim_F V = m$. a) m b) n c) m^2 d) n^2	K1	CO3
	6	Given that if W is a subspace of V and define $W^{\perp} = \{x \in V/(x, w) = 0 \; for \; all \; w \in W\}$. Then $W^{\perp}$ is said to be ________. a) orthogonal to W b) orthonormal to W c) Inner product space W d) orthogonal complement to W	K2	CO3
4	7	Write the row rank of a matrix is the dimension of its ________. a) Row Space b) Column space c) Vector Space d) Vector Subspace	K1	CO4
	8	Describe that if zero is a characteristic root of A if and only if A is ________. a) Non-singular b) Invertible c)Triangular d) Singular	K2	CO4
5	9	Given that A is an associative ring and it is also a vector space over F such that for all $a, b \in A$ and $\alpha \in F$, $\alpha(ab) = (\alpha a)b = a(\alpha b)$. Then A is called as ________. a) Algebra over F b) Subalgebra over F c) Inner product space over F d) Field over F	K1	CO4
	10	Predict that if $T, S \in A(V)$ and if S is regular, then T and STS^{-1} have the same ________. a) Characteristic polynomial b) Zero polynomial c) Maximum polynomial d) Minimum polynomial	K2	CO4

SECTION - B (35 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Show that if the matrix products AB and BC are defined, then $(AB)C = A(BC)$.	K3	CO1
		(OR)		

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1	11.b.	Show that any square matrix is expressible as a sum of symmetric matrix and a skew-symmetric matrix. Show further that this representation is unique.	K3	CO1
2	12.a.	Show that if V is the internal direct sum of $U_1, U_2, \ldots, U_n$, then V is isomorphic to the external direct sum of $U_1, U_2, \ldots, U_n$.	K3	CO2
		(OR)		
	12.b.	Show that $L(S)$ is a subspace of V.		
3	13.a.	Explain Schwarz inequality with proof.	K4	CO3
		(OR)		
	13.b.	(i) Explain orthogonal complement and orthonormal set of vectors. (ii) If $\{v_i\}$ is an orthonormal set, then the vectors in $\{v_i\}$ are linearly independent. If $w = \alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_n v_n$, then $\alpha_i = (w, v_i)$ for $i = 1,2,\ldots,n$.		
4	14.a.	Estimate the row reduced echelon matrix of the matrix $A = \begin{bmatrix} 0 & 2 & 4 & -4 \\ 2 & 3 & 2 & 4 \\ 1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 1 \end{bmatrix}$	K5	CO4
		(OR)		
	14.b.	Verify that the characteristic roots of a Hermitian matrix all are real.		
5	15.a.	Examine that if V is finite-dimensional over F, then $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not zero.	K4	CO4
		(OR)		
	15.b.	Examine that if V is finite-dimensional over F, then $S, T \in A(V)$. (i) $r(ST) \leq r(T)$; (ii) $r(TS) \leq r(T)$; (and so $r(ST) \leq min\{r(T), r(S)\}$) $iii)$ $r(ST) = r(TS) = r(T)$ for S regular in $A(V)$		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Determine the inverse of $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix}$.	K5	CO1
2	17	Investigate that if $v_1, v_2, \ldots, v_n$ is a basis of V over F and if $w_1, w_2, \ldots, w_m$ in V are linearly independent over F, then $m \leq n$.	K4	CO2
3	18	Verify that let V be a finite-dimensional inner product space; then V has an orthonormal set as a basis.	K5	CO3
4	19	Illustrate the following matrix to find the characteristic roots and characteristic vectors of the matrix: $A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & 3 \\ 3 & 2 & -3 \end{bmatrix}$.	K4	CO4
5	20	Analyse that if V is n -dimensional over F and if $T \in A(V)$ has the matrix $m_1(T)$ in the basis $v_1, v_2, \ldots, v_n$ and the matrix $m_2(T)$ in the basis $w_1, w_2, \ldots, w_n$ of V over F, then there is an element $C \in F_n$ such that $m_2(T) = C m_1(T) C^{-1}$.	K4	CO4

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