

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
DEGREE EXAMINATION DECEMBER
(First Semester)

Branch - MATHEMATICS

CALCULUS – I

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

$$(10 \times 1 = 10)$$

Module No.	Question No.	Question	K Level	CO
1	1	Find the domain of the vector function $r(t) = t^2, \sqrt{t-1}, \sqrt{5-t}$? a) [1,5] b) [2,5] b) [3,5] d) [4,5]	K1	CO1
	2	Find the length of the curve $r(t) = \langle 2\sin t, 5t, 2\cos t \rangle, -10 \leq t \leq 10$? a) $20\sqrt{29}$ b) $21\sqrt{29}$ c) $22\sqrt{29}$ d) $23\sqrt{29}$	K2	CO1
2	3	Find the value of $\lim_{(x,y) \rightarrow (1,2)} (x^2y^3 - x^3y^2 + 3x + 2y)$? a) 11 b) 12 c) 13 d) 14	K1	CO2
	4	Find the tangent plane to elliptic paraboloid $z=2x^2+y^2$ at the point (1,1,3) ? a) $4x+2y-3$ b) $2x+4y-3$ c) $4x-2y-3$ d) $2x-4y-3$	K2	CO2
3	5	Find y' if $x^3+y^3=6x$? a) $\frac{-(x^2-2y)}{y^2-2x}$ b) $\frac{x^2+2y}{y^2-2x}$ c) $\frac{x^2-2y}{y^2-2x}$ d) $\frac{x^2+2y}{y^2+2x}$	K1	CO3
	6	Use Langrange Multiplier to find maximum value of given function $f(x,y)=x^2+y^2+z^2$; $x^4+y^4+z^4=1$? a) $\sqrt{3}$ b) $\sqrt{2}$ c) $\sqrt{4}$ d) $\sqrt{5}$	K2	CO3
4	7	Find $\int_0^3 f(x,y)dx, f(x,y) = 2x + 3x^2y$? a) $9+27y$ b) $3+18y$ c) $9-27y$ d) $3-18y$	K1	CO4
	8	Find the value of iterated integral $\int_1^2 \int_y^{e^y} \sqrt{x} dx dy$? a) $\frac{4}{9}e^{3/2} - \frac{32}{45}$ b) $\frac{4}{9}e^{3/2} + \frac{32}{45}$ c) $\frac{4}{-9}e^{3/2} - \frac{32}{45}$ d) $-\frac{4}{9}e^{3/2} + \frac{32}{45}$	K2	CO4
5	9	Find the jacobian of the transformation $x=u+4v, y=3u-2v$? a) 14 b) -14 c) 13 d) -13	K1	CO5
	10	Use a triple integral to find the volume of the tetrahedron T bounded by the planes $x+2y+z=2, x=2y, x=0, z=0$? a) $1/2$ b) $1/3$ c) $-1/2$ d) $-1/3$	K2	CO5

Cont...

SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Find the unit normal and binormal vectors for the circular helix $r(t) = \cos t \mathbf{i} + \sin t \mathbf{j} + t \mathbf{k}$.	K2	CO1
		(OR)		
	11.b.	Find the vector equation and parametric equation for the line segment that joins the point P (1,3,-2) to the point Q (2,-1,3).		
2	12.a.	Verify the limit exists for given function $f(x) = \frac{xy}{x^2+y^2}$.	K3	CO2
		(OR)		
	12.b.	Show that $f(x) = xe^{xy}$ is differentiable at (1,0) and find its linearization. Then use it to approximate $f(1.1,-0.1)$		
3	13.a.	If $g(s,t) = f(s^2-t^2, t^2-s^2)$ and f is differentiable. Show that g satisfies the equation $t \frac{\partial g}{\partial s} + s \frac{\partial g}{\partial t} = 0$	K4	CO3
		(OR)		
	13.b.	Find the directional derivatives $D_u f(x,y)$ if $f(x,y) = x^3 - 3xy + 4y^2$ and u is the unit vector given by angle $\frac{\pi}{6}$. What is $D_u f(1,2)$?		
4	14.a.	Evaluate $\iint_R y \sin(xy) dA$, $R = [1,2] \times [0,\pi]$	K4	CO4
		(OR)		
	14.b.	Evaluate the iterated integral $\int_0^1 \int_x^1 \sin(y^2) dy dx$		
5	15.a.	Evaluate $\iiint_E z dv$ where E is the solid tetrahedron bounded by the four planes $x=0, y=0, z=0, x+y+z=1$	K4	CO5
		(OR)		
	15.b.	Evaluate $\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^2 (x^2 + y^2) dz dy dx$.		

SECTION -C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Determine the vector function that represents the curve of intersection of the cylinder $x^2+y^2=1$ and the plane $y+z=2$.	K3	CO2
2	17	Examine $\lim_{(x,y) \rightarrow (0,0)} \frac{3x^2y}{x^2+y^2}$ if it exists.	K3	CO2
3	18	Find the points on the sphere $x^2+y^2+z^2=4$ that are closest to and farthest from the point (3,1,-1).	K4	CO3
4	19	Find the volume of the solid that lies under the paraboloid $z=x^2+y^2$ above the xy plane and inside the cylinder $x^2+y^2=2x$.	K4	CO4
5	20	Evaluate the integral $\iint_R e^{(x+y)/(x-y)} dA$ where R is the trapezoidal region with vertices (1,0), (2,0), (0,-2), (0,-1).	K4	CO5