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22MAU514N

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2025
(Fifth Semester)

Branch – **MATHEMATICS**

REAL ANALYSIS

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	The g.l.b of the set $\left\{\pi+1,\ \pi+\frac{1}{2}, \pi+\frac{1}{3}\ \pi+\frac{1}{4}, \ldots\right\}$ is (a) π (b) $\pi+1$ (c) 0 (d) ∞	K1	CO1
	2	The vector space R^k with the inner product and norm defined is called (a) Euclidean space (b) Sub space (c) Linear space (d) None	K2	
2	3	An infinite subset of a countable set is_____ (a) Denumerable (b)Uncountable (c)Numerable (d) None	K1	CO2
	4	Every neighborhood is (a) closed set (b) open set (c) Neither closed nor open (d) None	K2	
3	5	Cantor set is (a) Uncountable (b) Countable (c) Denumerable (d) None	K1	CO3
	6	The set of all algebraic numbers is (a)Countable (b) Uncountable (c)Empty (d) None	K2	
4	7	The sequence $\left\{(-1)^{(n+1)}\ \frac{1}{n}\right\}_{n=1}^{\infty}$ converges to (a) 0 (b) -1 (c) 1 (d) ∞	K1	CO4
	8	The series $\sum_{n=1}^{\infty}\frac{1}{n}$ is______. (a) Convergent (b) Divergent (c) Absolutely convergent (d) None	K2	
5	9	Is it necessary for a uniformly continuous function to be continuous ? (a) No (b) Yes (c) Ambiguous (d) None	K1	CO5
	10	The range of a continuous mapping f of a connected subset of a metric space X in to an other metric space Y is (a) Compact (b) Not Compact (c) Connected (d) Not connected	K2	

Cont...

SECTION - B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Prove the following : (i) If $x \in R, y \in R$ and $x > 0$, then there is a positive integer n such that $nx > y$. (ii) If $x \in R, y \in R$ and $x < y$, then there exists a $p \in Q$ such that $x < p < y$.	K3	CO1
		(OR)		
	11.b.	State and prove Schwarz inequality.		
2	12.a.	Illustrate with an example to show that set of all integers is countable.	K4	CO2
		(OR)		
	12.b.	Prove the following: (i) A set E is open if and only if its complement is closed. (ii) A set F is closed if and only if its complement is open.		
3	13.a.	State and prove Weierstrass theorem.	K3	CO3
		(OR)		
	13.b.	Let P be a nonempty perfect set in R^k, then prove that P is uncountable.		
4	14.a.	Evaluate $\lim_{n\to\infty}(1+\frac{1}{n})^n$.	K4	CO4
		(OR)		
	14.b.	Prove Leibnitz theorem for alternating series.		
5	15.a.	Prove that continuous function of a continuous function is continuous.	K3	CO5
		(OR)		
	15.b.	Prove that a mapping f of a metric space X into a metric space Y is continuos on X if and only if $f^{-1}(Y)$ is open in X for every open set V in Y.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	For every real $x > 0$ and every integer $n > 0$ show that there is one and only one real y such that $y^n = x$.	K4	CO1
2	17	Prove that the countable union of countable sets is countable.	K5	CO2
3	18	Every k-cell is compact– Justify.	K4	CO3
4	19	State and prove the Ratio test.	K5	CO4
5	20	Prove that f is uniformly continuous on X ,where f is a continuous mapping of a compact metric space X into a metric space Y.	K3	CO5

Z-Z-Z END