22MAU307N

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2025
(Third Semester)

Branch - **MATHEMATICS**

PARTIAL DIFFERENTIAL EQUATIONS AND FOURIER TRANSFORMS

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 × 1 = 10)

Question No.	Question	K Level	CO
1	What is the term for the variable in a first order partial differential equation? a) independent variable b) Constant variable c) dependent variable d) parameter variable	K2	CO1
2	Which is the quasi- linear PDE of first order? a) $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=nz$ b) $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=z^2$ c) $P(z)\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=0$ d) $\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2=1$	K1	CO1
3	When $B^2>4AC$, the slopes of the curves are _____ a)Imaginary b) Undefined c)Complex d)Real	K2	CO2
4	Which of the following is linear three-dimensional heat equation? a) $u_t=k(u_{xx}+u_{yy}+u_{zz})$ b) $u_{xx}+u_{yy}+u_{zz}=0$ c) $u_{tt}=c^2(u_{xx}+u_{yy}+u_{zz})$ d) $u_t+uu_x=\mu u_{xx}$	K1	CO2
5	If a function $f(x)$ satisfies the condition $f(x)=f(x+p)$ for all x, then $f(x)$ is called _____ a) Periodic function b) Linear function c) Constant function d) Exponential function	K2	CO3
6	The Fourier sine series is_____ a) $a_0+\sum_{n=1}^{\infty}a_n\cos nx$ b) $a_0+\sum_{n=1}^{\infty}a_n\sin nx$ c) $\sum_{n=1}^{\infty}a_n\cos nx$ d) $\sum_{n=1}^{\infty}a_n\sin nx$	K1	CO3
7	The Fourier cosine integral is_____ a) $f(x)=\int_0^{\infty}B(w)\sin wxdw$ b) $f(x)=-\int_0^{\infty}B(w)\sin wxdw$ c) $f(x)=\int_0^{\infty}A(w)\cos wxdw$ d) $f(x)=-\int_0^{\infty}A(w)\cos wxdw$	K1	C^4
8	The Fourier coefficients are called as _____ a) amplitude spectrum b) Time domain signal c) Frequency response d) Phase shift	K2	CO4
9	What is the value of the thermal conductivity k when the material is homogeneous and the temperature is non-extreme? a) Constant b) Zero c) Variable d) Infinite	K2	CO5
10	Which of the following equation represents the heat equation? a) $\frac{\partial u}{\partial t}=c^2\nabla^2u$ b) $\frac{\partial^2u}{\partial t^2}=c^2\nabla^2u$ c) $\frac{\partial u}{\partial t}=\frac{1}{c^2}\nabla^2u$ d) $\frac{\partial^2u}{\partial t^2}=\frac{1}{c^2}\nabla^2u$	K1	CO5

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SECTION - B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Question No.	Question	K Level	CO
11.a.	Find the partial differential equation of the family of planes, the sum of whose x, y, z intercepts is equal to unity.	K3	CO3
	(OR)		
11.b.	Find the general integral of the linear PDE: $pz - qz = z^2 + (x+y)^2$		
12.a.	Reduce the equation $\sin^2(x)u_{xx} + \sin(2x)u_{xy} + \cos^2(x)u_{yy} = x$ to canonical form and hence solve it.	K2	CO2
	(OR)		
12.b.	Solve the PDE $(x^2D^2 + 2xyDD' + y^2D'^2)u = x^2y^2$.		
13.a.	Find the Fourier series of the function $f(x) = x + \pi$ if $-\pi < x < \pi$ and $f(x+2\pi) = f(x)$.	K4	CO4
	(OR)		
13.b.	State and prove orthogonality of the Trigonometric system.		
14.a.	Find the Fourier cosine and sine transforms of the function $f(x) = \begin{cases} k & if \quad 0 < x < a \\ 0 & if \quad x > a \end{cases}$	K3	CO3
	(OR)		
14.b.	Find the Fourier transform $F(e^{-ax})$ of $f(x) = e^{-ax}$ if $x > 0$ and $f(x) = 0$ if $x < 0$; here $a > 0$.		
15.a.	Find the temperature in the infinite bar if the initial temperature is $f(x) = \begin{cases} U_0 = const & if \quad \lvert x \rvert < 1 \\ 0 & if \quad \lvert x \rvert > 1 \end{cases}$	K4	CO4
	(OR)		
15.b.	Find the temperature $u(x,t)$ in a laterally insulated copper bar 80 cm long if the initial temperature is $100\sin(\pi x/80)^\circ C$ and the ends are kept at $0^\circ C$. How long will it take for the maximum temperature in the bar to drop to $50^\circ C$?		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Question No.	Question	K Level	CO
16	Compute the complete integral of $(p^2 + q^2)y = qz$.	K4	CO4
17	Find the real function $u(x,y)$, which reduces to zero when $y=0$ and satisfy the PDE $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -\pi(x^2 + y^2)$.	K4	CO4
18	Find the two half-range expansion of the function $f(x) = \begin{cases} \frac{2k}{L}x & if \quad 0 < x < \frac{L}{2} \\ \frac{2k}{L}(L-x) & if \quad \frac{L}{2} < x < L \end{cases}$	K4	CO4
19	State and Prove Convolution theorem.	K4	CO4
20	Find the temperature in an Infinite bar.	K4	CO4

Z-Z-Z END