TOTAL PAGES : 2
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PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2025
(Third Semester)

Branch - **COMPUTER SCIENCE WITH DATA ANALYTICS**

LINEAR ALGEBRA

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	A square matrix A is invertible if and only if ______. a) $det(A) \neq 0$ b) $det(A) = 0$ c) A is symmetric d) A is diagonal	K1	CO1
	2	In LU factorization, a matrix A is expressed as ______. a) $A = L + U$ b) $A = UL$ c) $A = LU$ d) $A = L^{-1}sU$	K2	CO1
2	3	Two vectors u and v are orthogonal if ______. a) $u.v = 1$ b) $u.v = 0$ c) $u + v = 0$ d) $u - v = 0$	K1	CO2
	4	The Gram-Schmidt process is used to ______. a) Find eigenvalues b) Compute determinants c) Solve linear systems d) Construct an orthogonal (or orthonormal) basis	K2	CO2
3	5	The determinant of a matrix is zero if and only if ______. a) The matrix is invertible b) The matrix is diagonal c) The matrix is singular d) The matrix is symmetric	K1	CO3
	6	The characteristic equation of A is obtained from ___. a) $det(A + \lambda I) = 0$ b) $A^T A = \lambda I$ c) $det(A) = 0$ d) $det(A - \lambda I) = 0$	K2	CO3
4	7	If T is a linear transformation and v is an eigenvector, then ____. a) $T(v) = 0$ b) $T(v) = v^T$ c) $T(v) = \lambda v$ d) $T(v) = \lambda^2 v$	K1	CO4
	8	If A is diagonalizable, then it can be written as ___. a) $A = PDP^{-1}$ b) $A = P + D + P^{-1}$ c) $A = PDP^T$ d) $A = P^T DP$	K2	CO4
5	9	Singular value Decomposition of a matrix A expresses it as ___. a) $A = U\Sigma V^T$ b) $A = PDP^{-1}$ c) $A = QR$ d) $A = LL^T$	K1	CO5
	10	The matrix of a linear transformation depends on ___. a) Only the dimension of the vector space b) The choice of basis for domain and codomain c) Only the determinant d) Only the eigenvalues	K2	CO5

SECTION - B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Prove that the dot product $v.w$ is zero when v is perpendicular to w.	K2	CO1
		(OR)		

Cont...

	11.b.	Multiply these matrices in the orders EF and FE: $E = \begin{bmatrix}1&0&0\\a&1&0\\b&0&1\end{bmatrix}$ $F = \begin{bmatrix}1&0&0\\0&1&0\\0&c&1\end{bmatrix}$	K2	CO1
2	12.a.	Find the LU factorization of A and the complete solution to $Ax = b$: $$A = \begin{bmatrix}1&3&1\\1&2&3\\2&4&6\\1&1&5\end{bmatrix},\quad b = \begin{bmatrix}1\\3\\6\\5\end{bmatrix} \text{ and then } b = \begin{bmatrix}1\\0\\0\\0\end{bmatrix}.$$	K2	CO2
		(OR)		
	12.b.	If $v_1, \dots, v_m$ and $w_1, \dots, w_n$ are both bases for the same vector space, then prove that $m = n$.		
3	13.a.	Prove that $A^T A$ is invertible if and only if the columns of A are linearly independent.	K3	CO3
		(OR)		
	13.b.	Explain the orthogonal matrix with three examples.		
4	14.a.	Compute the determinants of A and B from six terms. Are their columns independent? $$A = \begin{bmatrix}1&2&3\\1&0&1\\1&1&0\end{bmatrix} \text{ and } B = \begin{bmatrix}1&2&3\\4&4&4\\5&6&7\end{bmatrix}$$	K3	CO4
		(OR)		
	14.b.	Find the axes of the tilted ellipse $5x^2 + 8xy + 5y^2 = 1$.		
5	15.a.	Find the SVD of the singular matrix $$A = \begin{bmatrix}2&2\\1&1\end{bmatrix}.$$	K3	CO5
		(OR)		
	15.b.	T reflects every vector v across the straight line at angle θ. The output $T(v)$is the mirror image of v on the other side of the line. Find the matrix A in the standard basis and the matrix Λ in the eigenvector basis.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Find A^{-1} (if it exists) by Gauss Jordan elimination on [A I]: $$A = \begin{bmatrix}2&1&1\\1&2&1\\1&1&2\end{bmatrix} \quad A = \begin{bmatrix}2&-1&-1\\-1&2&-1\\-1&-1&2\end{bmatrix}$$	K3	CO1
2	17	Explain the four subspaces for U with examples.	K4	CO2
3	18	If $A = \begin{bmatrix}1&0\\1&1\\1&2\end{bmatrix}$ and $b = \begin{bmatrix}6\\0\\0\end{bmatrix}$ find $\hat{x}$ and p and P.	K3	CO3
4	19	Use Cramer's Rule (it needs four determinants) to solve $x_1 + x_2 + x_3 = 1, -2x_1 + x_2 = 0, \; -4x_1 + x_3 = 0.$	K4	CO4
5	20	Find the singular value decomposition of $A = \begin{bmatrix}2&2\\-1&1\end{bmatrix}.$	K3	CO5

Z-Z-Z END