22MAU519BN / 23MCU524BN

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2025
(Fifth Semester)

Common to Branches – **MATHEMATICS & MATHEMATICS WITH COMPUTER APPLICATIONS**

MAJOR ELECTIVE COURSE – I: NUMBER THEORY

Time: Three Hours | Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	According to Principle of Mathematical Induction, If P(n) is any statement involving natural numbers such that P(1) is true and P(r) = P(r^+) then _____ is true for all N. a) P(n) b) P(0) c) P(n-2) d) None	K1	CO1
	2	An integer a is said to be divisible by a non zero integer if there exits another integer c such that a = bc, then b is called _____ of a. a) Factor b) integral part c) imaginary part d) None	K2	CO1
2	3	A positive integer greater than 1 and not a ______ number is called composite number. a) Factor b) composite c) Prime d) None	K1	CO1
	4	If two adjacent integers are prime then they are called ________ twins. a) Identical b) Siamese c) different d) None	K2	CO1
3	5	According to Fermat's conjecture, the integer F_n = _______. a) 2^{2^n} b) $2^{2^n}+1$ c) n+1 d) None	K1	CO1
	6	Fermat's numbers are _____ . a) Prime b) Co-prime c) composite d) None	K2	CO1
4	7	If n is a prime number then = $(a+b)^n$= ______. a) $(a^n + b^n)\ mod\ n$ b) (a+b) modn c) mod n d) none	K1	CO1
	8	The congruence $x^2 \equiv 1(\text{ mod} p)$ has exactly two solutions namely 1 and ___. a) P-1 b) p c) -1 d) none	K2	CO1
5	9	Any three positive integers x,y,z such that $x^2 + y^2 = z^2$ are called as _____ triple. a) Fermat b) Newton c) Pythagorean d) None	K1	CO1
	10	Every prime P can be represented as a sum of _______ a) four squares b) two squares c) five cubes d) None	K2	CO1

Cont...

SECTION - B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	State and prove Trichotomy law.	K2	CO1
		(OR)		
	11.b.	Prove that $[a, b] = \frac{ab}{(a,b)}$ where a, b are integers and $[a, b]$ is LCM of a,b and (a, b) is HCF of a,b.		
2	12.a.	Prove that $\frac{n}{\varphi(n)} = \sum_{d/n} \frac{\mu^2(d)}{\varphi(d)}$.	K2	CO3
		(OR)		
	12.b.	Find the highest power of 7 dividing 1000!		
3	13.a.	Prove that Fermat numbers are co-primes.	K2	CO1
		(OR)		
	13.b.	Prove that $3^{4n+2} + 5^{2n+1} = M(14)$.		
4	14.a.	Prove that p! and (p-1)! – 1 are co-primes if p is an odd prime.	K3	CO4
		(OR)		
	14.b.	Solve 5x = 3(mod 24).		
5	15.a.	Find all the Pythagorean triples whose terms are in arithmetic progression.	K4	CO2
		(OR)		
	15.b.	Prove that $x^4 - y^4 = z^4$ has no solution in integers with $yz \neq 0$.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Prove by Mathematical induction that $3^{2n-1} + 2^{n+1} = \mu(7)$, where n∈ N.	K2	CO1
2	17	Verify that 220 and 284 are amicable numbers 220 = $2^2.5^1.11^1$	K2	CO3
3	18	Show that every number and its cube when divided by 6 leave the same remainder.	K3	CO4
4	19	Show that$16^{99} \equiv 1$ mod (437).	K4	CO2
5	20	Prove that if n > 1, each non negative primitive solution of $x^2 + y^2 = n$ determine a unique a modulo n such that ax ≡ -1 (mod n).	K3	CO4

Z-Z-Z END