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22STP313N

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2025
(Third Semester)

Branch- **STATISTICS**

STOCHASTIC PROCESSES

Time: Three Hours   Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks $(10 \times 1 = 10)$

Module No.	Question No.	Question	K Level	CO
1	1	What does it mean for a Markov chain to have stationary transition probabilities? a) The transition probabilities change over time. b) The transition probabilities depend only on the time step. c) The transition probabilities are independent of time. d) The transition probabilities depend on initial conditions.	K1	CO1
	2	What is meant by an absorbing state in a Markov chain? a) A state that can be left only once b) A state that cannot be left once it is entered c) A state with a periodic transition d) A transient state that appears early in the process	K2	CO1
2	3	Consider a two-state irreducible Markov chain with states 0 and 1 and transition matrix $P = \begin{pmatrix} p & 1-p \\ 1-q & q \end{pmatrix}$ Under what condition does this Markov chain have a stationary distribution? a) Only if $p + q < 1$ b) For any $p, q \in (0,1)$ c) Only if $p + q = 1$ d) Only if $p = q$	K1	CO1
	4	Which of the following ensures that the limiting distribution exists for an irreducible and positive recurrent Markov chain? a) All states must be absorbing b) The chain must be aperiodic c) All transition probabilities must be equal d) The chain must have a periodic structure with period $d > 1$	K2	CO1
3	5	For a random walk with absorbing barriers at 0 and N, starting at position k, what is the probability of absorption at N? a) k/N b) $N - k/N$ c) k/N^2 d) 1	K1	CO1
	6	What is the key difference between Kolmogorov backward and forward equations? a) The forward equation describes probabilities moving backward in time. b) The backward equation tracks probabilities starting from the final state. c) The backward equation deals with initial probabilities, while the forward equation tracks transitions over time. d) The forward equation only applies to discrete-time chains.	K2	CO1
4	7	In a Yule-Furry process, which models population growth, what is the expected population size at time t given that the process starts with a single individual and the birth rate is λ? a) λe^t b) $1 + \lambda t$ c) $1 + e^{\lambda t}$ d) $e^{\lambda t}$	K1	CO1
	8	For a Poisson process, what is the variance of the number of events N(t) occurring in a fixed time interval t? a) $\lambda^2 t$ b) t/λ c) λt d) $2\lambda t$	K2	CO1
5	9	Which of the following integral expressions gives the renewal density function f(t)? a) $f(t) = \int_0^t F(t-x)dx$ b) $f(t) = \frac{d}{dt}m(t)$ c) $f(t) = 1 - F(t)$ d) $f(t) = \lim_{n\to\infty} \frac{N(t)}{t}$	K1	CO1

Cont....

5	10	Wald's Decomposition Theorem states that for a sum $S_N = X_1 + X_2 + \cdots + X_N$, where N is a stopping time independent of the X_i, the expected sum $E[S_N]$ is: a) $E[S_N] = N\,E[X_1]$ b) $E[S_N] = E[N]\ E[X_1]$ c) $E[S_N] = E[N] + E[X1]$ d) $E[S_N] = E[N^2] \cdot E[X1]$	K2	CO1

SECTION - B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks $(5 \times 7 = 35)$

Module No.	Question No.	Question	K Level	CO
1	11.a.	Explain the Spatially homogeneous Markov chains with an example.	K2	CO2
		(OR)		
	11.b.	Define period of a state of a Markov chain and state three properties of it.		
2	12.a.	Define stationary distribution of a Markov Chain. Show that an irreducible Markov Chain has a stationary distribution if and only if it is positive recurrent.	K3	CO3
		(OR)		
	12.b.	Obtain the equivalence class(es) and check for the recurrence of states for the Markova chain with state space S= {1,2,3,4} and transition probability matrix. $$P = \begin{Vmatrix} 0 & 1 & 0 & 0 \\ 1/3 & 0 & 2/3 & 0 \\ 0 & 2/3 & 0 & 1/3 \\ 0 & 0 & 1 & 0 \end{Vmatrix}$$		
3	13.a.	Prove that the two-dimensional random walk is recurrent.	K3	CO4
		(OR)		
	13.b.	Define random walk and write down the transition probability matrix for a random walk with absorbing barrier, reflections barrier and elastic barrier.		
4	14.a.	Obtain $P_n(t)$ for the Yule process under the condition that X(0) = N = 1.	K4	CO5
		(OR)		
	14.b.	State the postulates and derive the Poisson process.		
5	15.a.	Show that the Poisson process can be viewed as a renewal process.	K4	CO4
		(OR)		
	15.b.	Explain Type I and Type II counters models in renewal process.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks $(3 \times 10 = 30)$

Module No.	Question No.	Question	K Level	CO
1	16	Explain with one example each the different types of stochastic process with respect to index parameter and state space.	K3	CO2
2	17	State and prove the theorem used to find a stationary probability distribution when the Markov chain is positive recurrent a periodic.	K3	CO3
3	18	For a birth and death process derive the forward and backward Kolmogorov differential equations.	K4	CO3
4	19	Consider a pure death process where $_n$ = n for n = 1,2,…,i.e. P{X(t+h) = j | X(t) = k} for j > k, t and k are positive. Assume an initial population of size i. Find $P_n(t) = P\{X(t) = n\}$, E{X(t)} and Var{X(t)}.	K4	CO4
5	20	State and prove the elementary renewal theorem.	K4	CO5

Z-Z-Z END