TOTAL PAGES : 2
22STP102N

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2025
(First Semester)

Branch - **STATISTICS**

ADVANCED PROBABILITY THEORY

Time: Three Hours | Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	If X and Y are independent variables then Cov(X,Y) =________ a) E(X)E(Y) b)E(XY) c)1 d)0	K1	CO1
	2	If Cis a constant then E(CX) = ___________ a) E(X) b)C c) C+E(X) d)CE(X)	K2	CO1
2	3	Characteristic function is affected by a) origin b) scale c) both origin and scale d)none of the above	K1	CO2
	4	The characteristic function will ________ if the moment generating function exist or not. a) always exists b) not always c) discrete distribution alone d) none of the above	K2	CO2
3	5	Tailed event of independent random variable have probability a)0 b)1 c)0 or 1 d) between 0 to 1	K1	CO3
	6	If $A_1, A_2, \ldots\ldots\ldots\ldots \in \mathfrak{F}$ then $\Sigma P(A_n)<\infty$ then $P\{A_n \text{ i.o}\}=$ a) ∞ b) -∞ c) 0 d)1	K2	CO3
4	7	Almost sure convergence implies convergence a)rth mean b)probability c)distribution d) none	K1	CO4
	8	If is a sequence of random variable and $n\to\infty$ and $\{X_n\}$ is said to converge to x in rth mean iff $E\|X_n-X\|^r\to$ a)0 b) -∞ c) 0 d)1	K2	CO4
5	9	The sample mean converges almost surely to the expected value as the sample size increases is a) Weak law of large number b) convergence in rth mean c) convergence in distribution d) Strong law of large number	K1	CO5
	10	The De Moivre-Laplace theorem states a)The sum of independent random variables approaches a uniform distribution. b) The sample mean of a sequence of independent Bernoulli trials approaches a normal distribution as the number of trials increases. c) The distribution of a sum of random variables converges to a Poisson distribution. d) The variance of the sample mean approaches infinity as the sample size increases.	K2	CO5

Cont...

SECTION – B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	State and prove Basic inequality.	K2	CO1
		(OR)		
	11.b.	The variance of X can be regarded as consisting of two parts, the expectation of the conditional variance and the variance of the conditional expectation.		
2	12.a.	Define Characteristic function and state its properties.	K2	CO2
		(OR)		
	12.b.	Find the density function whose characteristic function is given y c(t)= exp $(-1/2t^2)$		
3	13.a.	Define stochastic independence. And prove if X and Y with f(x,y) are independent iff f(x,y) can be expressed as the product of a non-negative function of x alone and none-negative function of Y alone.	K3	CO3
		(OR)		
	13.b.	State and prove Borel and Cantelli lemma.		
4	14.a.	Almost sure convergence is unique Justify the statement.	K2	CO4
		(OR)		
	14.b.	Prove that convergence in rth mean implies convergence of expectations in rth mean. ie, Prove $x_n \xrightarrow{r} x \Rightarrow E\|x_n^r\| \to E\|x^r\|$		
5	15.a.	Establish Bernoulli law of large numbers.	K5	CO5
		(OR)		
	15.b.	Prove Kolmogrov strong law of large numbers.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	State and prove Holder's inequality.	K4	CO2
2	17	Characteristic function uniquely determines the distribution. Justify the statement with proof.	K4	CO3
3	18	Establish Borel 0-1 Law and justify the statement of the Law.	K4	CO4
4	19	State and Prove Helly Bray Lemma.	K4	CO4
5	20	State and prove Lindeberg Levy theorem.	K5	CO5

Z-Z-Z END