

**PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)**

**MSc (SS) DEGREE EXAMINATION MAY 2025
(First Semester)**

Branch – SOFTWARE SYSTEMS (Five Years Integrated)

CALCULUS AND ITS APPLICATIONS

Time: Three Hours

Maximum: 75 Marks

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

(10 × 1 = 10)

| Question No. | Question | K Level | CO |
|--------------|---|---------|-----|
| 1 | Given $\sin^2 \theta = \frac{3}{4}$, then the angle θ is (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$ | K1 | CO1 |
| 2 | Given f is continuous at c and g is continuous at f(c), then $g \circ f$ is continuous at (a) c (b) f(c) (c) g(c) (d) f(g(c)) | K2 | CO1 |
| 3 | $\lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n$ is (a) e^x (b) e^{-x} (c) e^{nx} (d) e^{-nx} | K1 | CO2 |
| 4 | $\sum_{n=0}^{\infty} \frac{4}{2^n}$ is (a) 2 (b) 4 (c) 8 (d) 16 | K2 | CO2 |
| 5 | Given $f(x, y, z) = \frac{x-y}{y^2+z^2}$, then $f(0, -1/3, 0)$ is (a) 0 (b) 1 (c) 1/3 (d) 3 | K1 | CO3 |
| 6 | Given $f(x, y) = x \cos y + ye^x$, then $\frac{\partial^2 f}{\partial x \partial y}$ is (a) ye^x (b) $-x \cos y$ (c) $-\sin y + e^x$ (d) none of these | K2 | CO3 |
| 7 | Given $y' = 1 + y^2$ (a) $y = \sin(x + c)$ (b) $y = \cos(x + c)$ (c) $y = \tan(x + c)$ (d) $y = \cot(x + c)$ | K1 | CO4 |
| 8 | Given $y' = -2xy, y(0) = 1.8$ (a) $y = ce^{-x^2}$ (b) $y = ce^{-x^4}$ (c) $y = cx^2$ (d) $y = cx^4$ | K2 | CO4 |
| 9 | The period for $\cos 2x$ is (a) π (b) $\frac{\pi}{2}$ (c) 2π (d) none of these | K1 | CO5 |
| 10 | Given f(x) is an odd function with period 2π , then (a) $f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx$ (b) $f(x) = \sum_{n=1}^{\infty} a_n \cos nx$ (c) $f(x) = a_0 + \sum_{n=1}^{\infty} b_n \cos nx$ (d) $f(x) = \sum_{n=1}^{\infty} b_n \cos nx$ | K2 | CO5 |

SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks

(5 × 7 = 35)

| Question No. | Question | K Level | CO |
|--------------|---|---------|-----|
| 11.a. | Given $f(x) = 3x + 4, g(x) = 2x - 1$ and $h(x) = x^2$. Then find (i) $(f \circ g)(x)$ (ii) $(g \circ f)(x)$ (iii) $(f \circ f)(x)$ (iv) $(g \circ g)(x)$ (v) $(f \circ g \circ h)(x)$ | K1 | CO1 |
| (OR) | | | |
| 11.b. | Find the value of (i) $\lim_{x \rightarrow c} (x^3 + 4x^2 - 3)$ (ii) $\lim_{x \rightarrow c} \left(\frac{x^4 + x^2 - 1}{x^2 + 5} \right)$ (iii) $\lim_{x \rightarrow -2} \sqrt{4x^2 - 3}$ | | |

Cont...

| | | | |
|-------|---|----|-----|
| 12.a. | Investigate the convergent of the following series (i) $\sum_{n=0}^{\infty} \frac{2^{n+5}}{3^n}$ (ii) $\sum_{n=1}^{\infty} \frac{4^n n! n!}{(2n)!}$ | K2 | CO2 |
| | (OR) | | |
| 12.b. | Find the Taylor series and Taylor polynomial generated by $f(x) = \cos x$ at $x = 0$ | | |
| 13.a. | Show that $f(x, y) = \begin{cases} \frac{2xy}{x^2+y^2}, & (x, y) \neq (0,0) \\ 0 & (x, y) = (0,0) \end{cases}$ is continuous at every point except the origin. | K2 | CO3 |
| | (OR) | | |
| 13.b. | If $f(x, y) = \frac{2y}{y+\cos x}$, then find f_x and f_y as functions. Given resistors of R_1, R_2 and R_3 ohms are connected in parallel to make an R - ohms resistor, the value of R can be found from the equation $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$. Find the value of $\frac{\partial R}{\partial R_2}$ when $R_1=30, R_2 = 45$ and $R_3 = 90$ ohms. | | |
| 14.a. | Test the exactness of $\cos(x+y) dx + (3y^2 + 2y + \cos(x+y)) dy = 0$. If exact solve. | K3 | CO4 |
| | (OR) | | |
| 14.b. | Solve the Bernoulli equation $y' = Ay - By^2$ | | |
| 15.a. | Find the Fourier coefficients of the periodic function $f(x) = \begin{cases} -k & \text{if } -\pi < x < 0 \\ k & \text{if } 0 < x < \pi \end{cases}$ and $f(x+2\pi)=f(x)$. Functions of this kind occur as external forces acting on mechanical systems, electromotive forces in electric circuits, etc. (The value of at a single point does not affect the integral; hence we can leave undefined at and .) | K3 | CO5 |
| | (OR) | | |
| 15.b. | Find the Fourier series of the function. $f(x) = \begin{cases} 0 & \text{if } -2 < x < -1 \\ k & \text{if } -1 < x < 1 \\ 0 & \text{if } 1 < x < 2 \end{cases}$ $p=2L=4, L=2$ | | |

SECTION -C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks

(3 × 10 = 30)

| Question No. | Question | K Level | CO |
|--------------|---|---------|-----|
| 16 | Prove that $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$ | K2 | CO1 |
| 17 | Given $\sum a_n$ be a series with positive terms and $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \rho$, then prove that (i) the series converges if $\rho < 1$ (ii) the series diverges if $\rho > 1$ or ρ is infinite (iii) the test is inconclusive if $\rho = 1$ | K2 | CO2 |
| 18 | Find the point $P(x, y, z)$ on the plane $2x + y - z - 5 = 0$ that is closest to the origin. | K2 | CO3 |
| 19 | Suppose that in winter daytime temperature in a certain office building is maintained at 70°F. The heating is shut off at 10 P. M. and turned on again at 6 A. M. On a certain day the temperature inside the building at 2 A.M. was found to be 65°F. The outside temperature was 50°F at 10 P. M. and had dropped to 40°F by 6 A. M. What was the temperature inside the building when the heat was turned on at 6 A. M.? | K3 | CO4 |
| 20 | Find the two half - range expansions of the function $f(x) = \begin{cases} \frac{2k}{L}x & \text{if } 0 < x < \frac{L}{2} \\ \frac{2k}{L}(L-x) & \text{if } \frac{L}{2} < x < L \end{cases}$ | K3 | CO5 |