PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2025
(First Semester)

Branch – **MATHEMATICS**

MATHEMATICAL STATISTICS

Time: Three Hours | Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No	Question No	Question	K Level	CO
1	1	If E(X+c)=10 and E(X-c)=6 then the value of c is_____ (a) 1 (b) 2 (c) $\frac{1}{2}$ (d) 0	K1	CO1
	2	The value of $\phi(-t) =$ _______. (a) $\phi(t)$ (b) $-\frac{1}{\phi(t)}$ (c) $\overline{\phi(t)}$ (d) $-\overline{\phi(t)}$	K2	CO1
2	3	Mode of the binomial distribution $B\left(7,\frac{1}{2}\right)$ is_______ (a) 3 (b) 4 (c) 3 and 4 (d) 3/2	K1	CO2
	4	If X is a poisson variable with P(X=0)=P(X=1)=c then the value of c is _______ (a) e (b) $log\ e$ (c) $\log\left(\frac{1}{e}\right)$ (d) $\frac{1}{e}$	K2	CO2
3	5	The random variable X_n has an asymptotically normal distribution with mean is ________. (a) npq (b) np (c) $\sqrt{npq}$ (d) $\sqrt{np}$	K1	CO3
	6	The sequence of random variables $\{Z_n\} = \{X_n - X\}$ is stochastically convergent to_____. (a) 1 (b) $-\infty$ (c) ∞ (d) 0	K2	CO3
4	7	If j is transient, then as $n \to \infty$, $p_{jj}^{(n)} \to$ _____. (a) 1 (b) 0 (c) ∞ (d) -1	K1	CO4
	8	If the process is real and $m = 0, \sigma^2 = 1, then\ R(\tau)$ is called________function. (a) Correlation (b) Regression (c) Moment (d) Stationary	K2	CO4
5	9	The process of making estimates about the population parameter from a sample is called__________. (a) Statistical independence (b) Statistical inference (c) Statistical hypothesis (d) Statistical decision	K1	CO5
	10	Infer the value of f-statistic having a cumulative probability of 0.95. (a) 0.55 (b) 0.5 (c) 0.05 (d) 0.05	K2	CO5

Cont...

SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks (5 × 7 = 35)

Module No	Question No	Question	K Level	CO
1	11.a.	Compute the variance of the binomial distribution.	K2	CO1
		(OR)		
	11.b.	Show that the characteristic function of the sum of an arbitrary finite number of independent random variables equals the product of their characteristic functions.		
2	12.a.	The random variable X has the beta distribution with $p = q = 2$; hence its density $f(y)$ has the form $f(y) = \begin{cases} 0 & for\ y \leq \ \&\ y \geq 1, \\ \frac{\Gamma(4)}{\Gamma(2)\Gamma(2)} y(1-y) = 6y(1-y) & for 0 < y < 1 \end{cases}$. what is the probability that X is not greater than 0.2 ?	K2	CO2
		(OR)		
	12.b.	Obtain moments of the gamma distribution.		
3	13.a.	A box contains a collection of IBM cards corresponding to the workers from some branch of industry. Of the workers 20% are minors & 80% adults. We select one IBM card in a random way & mark the age given on this card. Before choosing the next card, we return the first one to the box, so that the probability of selecting the card corresponding to a minor remains 0.2 we observe n cards in this manner. What value should n have in order that the probability will be 0.95 that the frequency of cards corresponding to minors lies between 0.18 & 0.22?	K3	CO3
		(OR)		
	13.b.	Apply Bernoulli's Law of large number Examine that - The sequence of random variables $\{X_n\}$ given by $P\left(Y_n = \frac{r}{n}\right) = \binom{n}{r} p^r (1-p)^{n-r}$ & $X_n = Y_n - p$ is stochastically convergent to 0, that is, for any $\varepsilon > 0$ we have $\lim_{n\to\infty} P(\lvert X_n \rvert > \varepsilon) = 0$.		
4	14.a.	Analyze the statement "the solutions $V_m(t)$ of the system $V_0'(t) = -\lambda_l V_0(t)$, $V_m'(t) = -\lambda_{l+m} V_m(t) + \lambda_{l+m-1} V_{m-1}(t)$ $(m = 1,2, \ldots)$ with the initial conditions $V_m(0) = \begin{cases} 1 & for\ m = 0, \\ 0 & for\ m \neq 0. \end{cases}$ satisfy the relation $\sum_{m=0}^{\infty} V_m(t) = 1$ iff $\sum_{m=0}^{\infty} \frac{1}{\lambda_{l+m}} = \infty$".	K4	CO4
		(OR)		
	14.b.	Analyze the statement "A process stationary in the wide sense is continuous iff its covariance function $R(\tau)$ is continuous at zero".		
5	15.a.	The random variables $X_k (k = 1, \ldots, 8)$ are independent & have the same normal distribution $N(0; 2)$. We consider the statistic $\chi^2 = \sum_{k=1}^{8} X_k^2$. Here the random variable χ^2 has eight degrees of freedom. The expected value & the standard deviation of this random variable are, respectively $m_1 = 32, \sqrt{\mu_2} = 16$. Compute the probability that χ^2 will exceed or equal 40.	K5	CO5
		(OR)		
	15.b.	Prove that, the sequence $\{F_n(t)\}$ of distribution functions of student's t with n degrees of freedom satisfies for every t the relation $\lim_{n\to\infty} F_n(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} e^{-t^2/2}\, dt$.		

SECTION -C (30 Marks)
Answer **ANY THREE** questions
ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No	Question No	Question	K Level	CO
1	16	Find the density function of the random variable X, whose characteristic function is $\phi_1(t) = \begin{cases} 1-\lvert t\rvert & for\ \lvert t\rvert \le 1, \\ 0 & for\ \lvert t\rvert > 1 \end{cases}$.	K1	CO1
2	17	Let the random variable x_n have a binomial distribution defined by the formula $P(x_n = r) = \frac{n!}{r!(n-r)!}p^r(1-p)^{n-r}$, where r takes on the values 0,1,2,…,n. if for $n = 1,2, \ldots$ the relation $p = \frac{\lambda}{n}$ holds where $\lambda > 0$ is a constant, then show that $\lim_{n\to\infty} P(X_n = r) = \frac{\lambda^r}{r!}e^{-\lambda}$.	K2	CO2
3	18	State and prove The Lindberg-Levy Theorem.	K3	CO3
4	19	Analyze the statement "A stochastic process $\{X_t, 0 \le t < \infty\}$, where X_t is the number of signals in the interval [0,t), satisfying conditions I to III & the equality $P(X_0 = 0) = 1$, is a homogeneous Poisson process".	K4	CO4
5	20	The random variables $X_k (k = 1, \ldots, 16)$ are independent & have the same density $f(x) = \frac{1}{2\sqrt{2\pi}} exp\left\{-\frac{1}{2}.\frac{(x-1)^2}{4}\right]$. Determine the distribution of (i) $\bar{x} = \frac{1}{16}\sum_{k=1}^{16} X_k$. (ii) $P(0 \le \bar{X} \le 2)$ (iii) $P(0 \le X \le 2)$	K5	CO5

Z-Z-Z END