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PSG COLLEGE OF ARTS & SCIENCE

(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2025

(IV [redacted] Semester)

Branch - **MATHEMATICS**

MATHEMATICAL METHODS

Time: Three Hours — Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	$g(s) = f(s) + \lambda \int_a^s e^{s-t} g(t)dt$ is ______ integral equation. a) Fredholm b) Volterra c) singular d) none of these	K1	CO1
	2	The inhomogeneous Fredholm integral equation with a separable kernel has __ solution. a) one b) more than one c) one and only one d) none of these	K2	CO1
2	3	If a Fredholm integral equation $g(s) = f(s) + \lambda \int_a^b \Gamma(s,t;\lambda) f(t)dt$ then the resolvent kernel $\Gamma(s,t;\lambda)$ satisfies the integral equation ______. a) $\Gamma(s,t;\lambda) = K(s,t) + \lambda \int_a^b K(s,x)\Gamma(s,t;x)dx$ b) $\Gamma(s,t;x) = K(s,t) + \lambda \int_a^b K(s,x)\Gamma(s,t;\lambda)dx$ c) $\Gamma(s,t;\lambda) = K(s,x) + \lambda \int_a^b K(s,t)\Gamma(s,t;x)dx$ d) $\Gamma(s,t;\lambda) = \Gamma(s,t;\lambda) + \lambda \int_a^b K(s,x)\Gamma(s,t;x)dx$	K1	CO1
	4	The series for the resolvent kernel $\Gamma(s,t;\lambda) = \sum_{m=1}^{\infty} \lambda^{m-1} K_m(s,t)$ is _____ convergent for all values of s and t in the circle $\lvert\lambda\rvert < B^{-1}$. a) uniformly b) absolutely c) both (a) and (b) d) none of these	K2	CO2
3	5	The boundary value problems for ordinary differential equations can be converted to integral equations of _____ type. a) Fredholm b) Volterra c) Abel d) homogeneous	K1	CO1
	6	A Fredholm integral equation with a kernel of the type $K(s,t) = \frac{H(s,t)}{\lvert t-s\rvert^{\alpha}}$, $0 < \alpha < 1$ where $H(s,t)$ is a bounded function then the kernel is ___. a) singular b) weakly singular c) weak d) none of these	K2	CO2
4	7	The curves $y = y(x)$ and $y = y_1(x)$ are close in the sense of _____ proximity if the absolute value of the difference $y(x) - y_1(x)$ and $y'(x) - y_1'(x)$ are small. a) zero order b) first order c) kth order d) none of these	K1	CO1
	8	The integral curves of Euler's equation are called ______. a) admissible curves b) parametric curves c) extremals d) externals	K2	CO1
5	9	$\frac{\partial v}{\partial x} + H\left(x, y_s, \frac{\partial v}{\partial y_s}\right) = 0$ is called ____ equation. a) Euler b) Jacobi c) Euler- Jacobi d) Hamilton-Jacobi	K1	CO1
	10	If a proper field is formed by a family of extremals of a certain variational problem, then it is called ___ field. a) slope of the b) extremal c) central d) external	K2	CO2

Cont...

SECTION – B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Explain the special cases of the equation $h(s)g(s) = f(s) + \lambda \int_a^s K(s,t)g(t)dt$.	K3	CO2
		(OR)		
	11.b.	Solve the homogeneous Fredholm integral equation $g(s) = \lambda \int_0^1 e^s e^t g(t)dt$.		
2	12.a.	Solve the integral equation $g(s) = f(s) + \lambda \int_0^1 e^{s-t} g(t)dt$.	K3	CO2
		(OR)		
	12.b.	Solve the Volterra equation $g(s) = 1 + \int_0^s st\, g(t)dt$.		
3	13.a.	Reduce the boundary value problem $y''(s) + \lambda P(s)y = Q(s)$, $y(a) = 0$, $y(b) = 0$ to a Fredholm integral equation.	K4	CO3
		(OR)		
	13.b.	Solve the integral equation $s = \int_0^s \frac{g(t)dt}{(s-t)^{1/2}}$.		
4	14.a.	State and prove the fundamental lemma of the calculus of variations.	K5	CO4
		(OR)		
	14.b.	Determine the extremal of the functional $v[y(x)] = \int_0^{\frac{\pi}{2}} (y''^2 - y^2 + x^2)dx$ that satisfies the conditions $y(0) = 1$, $y'(0) = 0$, $y\left(\frac{\pi}{2}\right) = 0$, $y'\left(\frac{\pi}{2}\right) = -1$.		
5	15.a.	Write the conditions that will be sufficient for a functional v to achieve an extremum on the curve C.	K4	CO3
		(OR)		
	15.b.	Test for an extremum the functional $v[y(x)] = \int_0^a y'^3\, dx$; $y(0) = 0$, $y(a) = b$, $a > 0$, $b > 0$.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Show that the integral equation $g(s) = f(s) + (1/\pi) \int_0^{2\pi} [\sin(s+t)]\, g(t)dt$ possesses no solution for $f(s) = s$, but that it possesses infinitely many solutions when $f(s) = 1$.	K4	CO3
2	17	Solve the Fredholm integral equation $g(s) = 1 + \lambda \int_0^1 (1 - 3st) g(t)dt$ and evaluate the resolvent kernel.	K4	CO3
3	18	Reduce the initial value problem $y''(s) + A(s)y'(s) + B(s)y = F(s)$, $y(a) = q_0$, $y(b) = q_1$ to Volterra integral equation.	K5	CO4
4	19	State and solve the brachistochrone problem.	K4	CO3
5	20	Test for an extremum the functional $\int_0^a (6y'^2 - y'^4 + yy')dx$, $y(0) = 0$, $y(a) = b$, $a > 0$, $b > 0$ in the class of continuous functions with continuous first derivative.	K5	CO4

Z-Z-Z END