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PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2025
(Fourth Semester)

Branch- MATHEMATICS

CONTROL THEORY

Time: Three Hours | Maximum: 75 Marks

SECTION-A (10 Marks)
Answer ALL questions
ALL questions carry EQUAL marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	$(e^A)^{-1} =$ (a) e^A (b) e^{-A} (c) e^1 (d) e^{-1}	K1	CO1
	2	If the system is observable at every $t \in I$ it is called ________ (a) convergent (b) partially observable (c) divergent (d) completely obervable	K2	CO1
2	3	The linear control system $\dot{x} = A(t)x + B(t)u$ is controllable on $[0, T]$ if for every pair of vectors $x_0, x_1 \in R^n$, there is a control $u \in L_m^2[0, T]$ such that the solution $x(t)$ of $\dot{x} = A(t)x + B(t)u$ satisfies ________. (a) $x(1) = x_0$ and $x(T) = x_1$ (b) $x(0) = 0$ and $x(T) = x_1$ (c) $x(0) = x_0$ and $x(T) = x_1$ (d) $x(0) = 0$ and $x(T) = 0$	K1	CO2
	4	System $\dot{x}(t) = A(t)x(t) + B(t)u(t) + f(t, x(t), u(t))$ is said to be completely controllable if for every $x_0, x_1 \in R^n$ there exists a continuous control function $u(t)$ defined on I such that the solution of the above equation satisfies ____________. (a) $x(1) = x_0$ and $x(T) = x_1$ (b) $x(0) = 0$ and $x(T) = x_1$ (c) $x(0) = 0$ and $x(T) = 0$ (d) $x(0) = x_0$ and $x(T) = x_1$	K2	CO2
3	5	Eigen values of the matrix $\begin{bmatrix} 1 & 5 \\ 5 & 1 \end{bmatrix}$ are (a) One Positive, one Negative (b) both are positive (c) both are negative (d) both are zero	K1	CO3
	6	The solution of $\phi(t)$ is called _____ if it is not stable. (a) un stable (b) complete stable (c) strong stable (d) uniform stable	K2	CO3
4	7	The system $\dot{x} = Ax + Bu, x \in R^n, u \in R^m$ is called an ______ (a) open loop system (b) closed loop system (c) circuit loop system (d) closed circuit loop system	K1	CO4
	8	The linear time invariant control system ___________ $\dot{x} = Ax + Bu, x \in R^n, u \in R^m$ is stabilizable if there exists an $m \times n$ Matrix K such that ___is stablity matrix. (a) $A - BK$ (b) $A + BK$ (c) $AB - K$ (d) $AB + K$	K2	CO4
5	9	In the cost functional equation, $Q(t)$ is ___ matrix (a) an $m \times n$ symmetric positive semidefinite (b) an $n \times n$ symmetric positive semidefinite (c) an $n \times m$ symmetric positive semidefinite (d) an $n \times n$ symmetric positive definite	K1	CO5
	10	$u(t) = G(t)x(t), t \in [0, T]$, where $G(t)$ is an ______ (a) $n \times n$ matrix valued function (b) $1 \times n$ matrix valued function (c) $m \times 1$ matrix valued function (d) $m \times n$ matrix valued function	K2	CO5

SECTION - B (35 Marks)
Answer ALL questions
ALL questions carry EQUAL Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Solve the initial value problem $\dot{x} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x, x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$	K3	CO1
		(OR)		
	11.b.	Organize: The constant coefficient system $\dot{x} = Ax, \dot{y} = Hx$ is observable on an arbitrary interval $[0, T]$ if and only if for some $k, 0 < k \leq n$ the rank of the observability matrix $rank \begin{bmatrix} H \\ HA \\ \vdots \\ HA^{k-1} \end{bmatrix} = n$		

Cont....

2	12.a.	Analyze: The system $\dot{x} = A(t)x + B(t)u$ is controllable on $[0, T]$ if and only if for each vector $x_1 \in R^n$ there is a control $u \in L_m^2[0, T]$ which steers 0 to x_1 during $[0, T]$.	K4	CO2
		(OR)		
	12.b.	Analyze: If rank B $= n$ then the system $\dot{x} = Ax + Bu$ is controllable.		
3	13.a.	Explain Gronwall's Inequality.	K2	CO3
		(OR)		
	13.b.	Explain: Let $X(t)$ be a fundamental matrix of the system $\dot{x}(t) = A(t)x(t)$. Assume that there exists a constant $K > 0$ such that $\int_0^t \lVert X(t,s)\rVert ds \leq K, t \geq 0$ then there exists a constant $M > 0$ such that $\lVert X(t)\rVert \leq Me^{-(\frac{1}{K})t}, t \geq 0$.		
4	14.a.	Suppose there are $m \times n$ matrices K_1, K_2 such that $(A + BK_1)$ and $-(A + BK_2)$ are stability matrices. Then examine the system $\dot{x} = Ax + Bu, x \in R^n, u \in R^m$ is controllable.	K4	CO4
		(OR)		
	14.b.	Prove that the linear control system $\dot{x} = Ax + Bu$ is stabilizable if and only if after reduction to the form A_2 is a stability matrix.		
5	15.a.	If $u(t) = -R^{-1}(t)B^*(t)K(t)x(t)$, then construct that J attains a local minimum.	K3	CO5
		(OR)		
	15.b.	Construct: For the continuous non linear system $\dot{x}(t) = A(t)x(t) + B(t)u(t) + f(t, x(t))$ with quadratic performance criteria $J = \frac{1}{2}x^*(T)Fx(T) + \frac{1}{2}\int_0^T [x^*(t)Q(t)x(t) + u^*(t)R(t)u(t)]dt$ the optimal control exists if $\lVert f(t,x) - f(t,y)\rVert \leq a\lVert x - y\rVert$ where a is positive constant, and is given by $u(x(t), t) = -R^{-1}(t)B^*(t)K(t)x(t) - R^{-1}(t)B^*(t)h(t,x)$ Where $K(t)$ satisfies the Riccati equation and $h(t,x) = -[A^*(t) - K(t)B(t)R^{-1}(t)B^*(t)]h(t,x) - K(t)f(t,x(t))$ $h(T,x) = 0$		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Examine: The equation $\dot{x}(t) = f(t,x), x(t_0) = x_0$ has a unique solution defined on $[t_0, t_0 + h], h > 0$ if the function $f(t,x)$ is continuous in the strip $t_0 \leq t \leq t_0 + h, \lvert x\rvert < \infty$ and satisfies the Lipschitz condtion. $\lvert f(t,x_1) - f(t,x_2)\rvert \leq K\lvert x_1 - x_2\rvert$ Where $K > 0$ is a constant.	K4	CO1
2.	17	Examine: Suppose the system $\dot{x} = A(t)x + B(t)u$ is completely controllable and the continuous function f is bounded locally in u (for $(t,x) \in I \times R^n$) and satisfies the following conditions (i) $\lim_{\lvert u\rvert \to \infty} \frac{\lvert f(t,x,u)\rvert}{\lvert u\rvert} = 0$ uniformly in $(t,x) \in I \times R^n$ (ii) for each $r > 0$ there exists a constant L such that for every $t \in I, x \in R^n, \lvert u\rvert \leq r$ we have $\lvert f(t,x,u)\rvert \leq L\lvert x\rvert$ Then the system $\dot{x}(t) = A(t)x(t) + B(t)u(t) + f(t, x(t), u(t))$ is controllable.	K4	CO2
3	18	Examine: Let $X(t)$ be a fundamental matrix of $\dot{x}(t) = A(t)x(t)$ such that $\int_0^t \lVert X(t,s)\rVert ds \leq K, t \geq 0$ Where $K > 0$ is a constant. Further let $\lVert f(t,x)\rVert = \mu\lVert x\rVert$ with $0 \leq \mu \leq 1/K$. 'Then the zero solution of of $\dot{x}(t) = A(t)x + f(t,x)$ is asymptotically stable.	K4	CO3
4	19	Examine: If the system $\dot{x} = Ax + Bu, x \in R^n, u \in R^m$ is controllable, then it is stabilizable.	K4	CO4
5	20	Examine: Given the linear system $\dot{x}(t) = A(t)x(t) + B(t)u(t)$ and the cost functional $J = \frac{1}{2}x^*(T)Fx(T) + \frac{1}{2}\int_0^T [x^*(t)Q(t)x(t) + u^*(t)R(t)u(t)]dt$ there exists an optimal control of the form $u(t) = -R^{-1}(t)B^*(t)K(t)x(t)$ Where $K(t)$ is the solution of the matrix Riccati equation with $K(T) = F$	K4	CO5

Z-Z-Z END