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PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
MSc DEGREE EXAMINATION MAY 2025
(Second Semester)
Branch- **MATHEMATICS**
COMPLEX ANALYSIS

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	If $ad - bc = 1$ in a linear transformation $w = \frac{az+b}{cz+d}$, then it said to be (a) translation (b) normalized (c) rotation (d) inversion	K1	CO1
	2	The index of a point with respect to the closed curve γ is denoted by $n(\gamma, a)$. It is given by the formula (a) $2\pi i \int_\gamma \frac{dz}{z-a}$ (b) $\frac{2}{\pi i}\int_\gamma \frac{dz}{z-a}$ (c) $\frac{1}{2\pi i}\int_\gamma \frac{dz}{z-a}$ (d)) $2\pi a \int_\gamma \frac{dz}{z-a}$	K2	CO1
2	3	"Given γ is a homologous to zero in Ω and $n(\gamma, z)$ is either 0 or 1 for any point z not on γ. If $f(z)$ and $g(z)$ are analytic in Ω and satisfy the inequality $\|f(z) - g(z)\| < \|f(z)\|$ on γ. Then $f(z)$ and $g(z)$ have the same number of zeros enclosed by γ" this is known as (a) Morera's theorem (b) Abel's theorem (c) Rouche's theorem (d) Cauchy's theorem	K1	CO2
	4	If u_1 and u_2 are harmonic in a region Ω, then (a) $\int_\gamma u_1 * du_1 - u_2 * du_2 = 0$ (b) $\int_\gamma u_1 * du_2 - u_2 * du_1 = 0$ (c) $\int_\gamma u_2 * du_2 - u_1 * du_1 = 0$ (d) $\int_\gamma u_2 * du_1 - u_1 * du_2 = 0$	K2	CO2
3	5	$log(1+z) =$ ______ (a) $1 + z + \frac{z^2}{2} + \cdots$ (b) $1 - \frac{z^2}{2} + \frac{z^4}{4} + \cdots$ (c) $z - \frac{z^2}{2} + \frac{z^3}{3} + \cdots$ (d) none of these	K1	CO3
	6	Every function which is meromorphic in the whole plane is the _______ of two entire function. (a) sum (b) product (c) quotient (d) composition	K2	CO3
4	7	Given $\varphi(t)$ is an analytic arc. If it is regular, then (a) $\varphi(t) = 0$ (b) $\varphi(t) \neq 0$ (c) $\varphi'(t) = 0$ (d) $\varphi'(t) \neq 0$	K1	CO4
	8	Harnack's inequalities valid only for _________ harmonic functions. (a) positive (b) negative (c) both (a) and (b) (d) none of these	K2	CO4
5	9	e^z has a period _______ (a) π (b) πi (c) 2π (d) $2\pi i$	K1	CO5
	10	The equation $\eta_1\omega_2 - \eta_2\omega_1 = 2\pi i$ known as (a) Weierstrass relation (b) Legendre relation (c)elliptic relation (d) canonical relation	K2	CO5

Cont...

SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Prove that the cross ratio (z_1, z_2, z_3, z_4) is real if and only if the four points lie on a circle or on a straight line.	K3	CO1
		(OR)		
	11.b.	Evaluate (i) $\int \frac{e^z}{z} dz$ on $\lvert z\rvert = 1$ (ii) $\int \frac{dz}{z^2+1} dz$ on $\lvert z\rvert = 2$.		
2	12.a.	Apply the method of residue to evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{a+\sin^2 x}$ where $\lvert a\rvert > 1$.	K5	CO2
		(OR)		
	12.b.	Given $f(z)$ is meromorphic in Ω with the zeros a_j and the poles b_k, then prove that $\frac{1}{2\pi i}\int_\gamma \frac{f'(z)}{f(z)} dz = \sum_j n(\gamma, a_j) - \sum_k n(\gamma, b_k)$ for every cycle γ, which is homologous to zero in Ω and does not pass through any of the zeros or poles.		
3	13.a.	Explain necessary and sufficient condition for absolute convergence of the product $\prod_1^\infty (1 + a_n)$ is the convergence of the series $\sum_1^\infty \lvert a_n\rvert$.	K5	CO3
		(OR)		
	13.b.	Construct the canonical product for $\sin \pi z$.		
4	14.a.	Given f is a topological mapping of a region Ω. If $z(t)$ tends to the boundary of Ω, then prove that $f(z(t))$ tends to the boundary of Ω'.	K5	CO4
		(OR)		
	14.b.	A continuous function $u(z)$ with mean-value property is necessarily harmonic.		
5	15.a.	Prove that any two bases of the same module are connected by unimodular transformation.	K5	CO5
		(OR)		
	15.b.	(i) Prove that the sum of the residues of an elliptic function is zero. (ii) Prove that an elliptic function without poles is a constant.		

SECTION -C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Suppose that $\varphi(\xi)$ is a continuous on the arc γ. Then show that the function $F_n(z) = \int \frac{\varphi(\xi)\, d\xi}{(\xi - z)^n}$ is analytic in each of the regions determined by γ and its derivative $F_n' = nF_{n+1}(z)$.	K3	CO1
2	17	State and prove Schwarz's lemma.	K5	CO2
3	18	Derive Jensen's formula and Poisson Jensen's formula.	K5	CO3
4	19	State and prove Reimann mapping theorem.	K5	CO3
5	20	Construct a basis (ω_1, ω_2) such that ratio $\tau = \frac{\omega_2}{\omega_1}$ satisfies the following conditions: (i) $Im\, \tau > 0$ (ii) $-\frac{1}{2} < Re\, \tau \leq \frac{1}{2}$ (iii) $\lvert\tau\rvert \geq 1$ (iv) $Re\, \tau \geq 0$ if $\lvert\tau\rvert = 1$ and show that the ratio is uniquely determined by these conditions and there is a choice of two four or six corresponding bases.	K5	CO3

Z-Z-Z END