**22MAP208N**

**PSG COLLEGE OF ARTS & SCIENCE**
(AUTONOMOUS)

**MSc DEGREE EXAMINATION MAY 2025**
(Second Semester)

Branch- **MATHEMATICS**

## TOPOLOGY

Time: Three Hours — Maximum: 75 Marks

### SECTION-A (10 Marks)

Answer **ALL** questions
**ALL** questions carry **EQUAL** marks (10 × 1 = 10)

| Module No. | Question No. | Question | K Level | CO |
|---|---|---|---|---|
| 1 | 1 | For X = {a, b, c, d}, which of the following is not a topology on X :<br>(a) $\tau = \{\varphi, \{a\}, \{b\}, \{a, b\}, X\}$<br>(b) $\tau = \{\varphi, \{b\}, \{c\}, \{b, c\}, \{c, d\}, \{b, c, d\}, X\}$<br>(c) $\tau = \{\varphi, \{b\}, \{c\}, \{a, c\}, \{b, c\}, \{b, c, d\}, X\}$<br>(d) $\tau = \{\varphi, \{a\}, X\}$ | K1 | CO1 |
| | 2 | A subset A of a topological space X is said to be closed if the set $X - A$ is -------<br>(a) Open (b) Closed (c) Limit point (d) Interior | K2 | CO1 |
| 2 | 3 | If $f: X \to Y$ and $g : Y \to Z$ are continuous, then the map $g \circ f : X \to Z$ is<br>(a) Open (b) continuous. (c) Limit point (d) Interior | K1 | CO2 |
| | 4 | A metric space is a space X together with a specific metric d that gives the topology of X.<br>(a) Metrizable (b) Standard bounded metric<br>(c) Product topology (d) Euclidean metric | K2 | CO2 |
| 3 | 5 | A space X is ---------if and only if the only subsets of X that are both open and closed in X are the empty set and X itself.<br>(a) subspace (b) closure<br>(c) Product topology (d) connected | K1 | CO3 |
| | 6 | A finite cartesian product of connected spaces is ------------<br>(a) Connected (b) uniform topology<br>(c) disconnected (d) T-shaped spaces | K2 | CO3 |
| 4 | 7 | A space X is said to be ------if every infinite subset of X has a limit point.<br>(a) Limit point compact (b) continuous<br>(c) Limit point (d) Interior | K1 | CO4 |
| | 8 | A subset A of a space X is said to be -------- in X if $\bar{A} = X$.<br>(a) Open (b) Closed (c) dense (d) Interior | K2 | CO4 |
| 5 | 9 | Every metrizable space is -----<br>(a) normal (b) Closed (c) dense (d) regular | K1 | CO5 |
| | 10 | Subspace of a regular space is ----------<br>(a) normal (b) Closed (c) dense (d) regular | K2 | CO5 |

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## SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks (5 × 7 = 35)

| Module No. | Question No. | Question | K Level | CO |
|---|---|---|---|---|
| 1 | 11.a. | Prove that If A is a subspace of X and B is a subspace of Y, then the product topology on A × B is the same as the topology A × B inherits as a subspace of X × Y. | K3 | CO1 |
| | | (OR) | | |
| | 11.b. | Prove that Let A be a subset of the topological space X. $A'$ be the set of all limit points of A. Then $\bar{A} = A \cup A'$. | K4 | |
| 2 | 12.a. | State and prove Sequence lemma. | K1 | CO2 |
| | | (OR) | | |
| | 12.b. | State and prove Pasting lemma. | K6 | |
| 3 | 13.a. | State and Prove Intermediate value theorem. | K1 | CO3 |
| | | (OR) | | |
| | 13.b. | State and Prove Uniform continuity theorem. | K5 | |
| 4 | 14.a. | Prove that Compactness implies limit point compactness, but not conversely. | K4 | CO4 |
| | | (OR) | | |
| | 14.b. | Prove that A subspace of a Hausdorff space is Hausdorff; a product of Hausdorff spaces is Hausdorff. | K4 | |
| 5 | 15.a. | State and pove Tychonoff theorem. | K1 | CO5 |
| | | (OR) | | |
| | 15.b. | State and pove Tietze extension theorem. | K1 | |

## SECTION -C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks (3 × 10 = 30)

| Module No. | Question No. | Question | K Level | CO |
|---|---|---|---|---|
| 1 | 16 | Let X be a topological spaces then the following condition are hold<br>(i) $\phi$ and X are closed<br>(ii) Arbitrary intersection of closed sets are closed<br>(iii) Finite union of closed set are closed | K2 | CO1 |
| 2 | 17 | Let X and Y be topological spaces and let $f$:X→Y. Then prove that the following are equivalent.<br>(i) $f$ is continuous.<br>(ii) For every subset A of X $f(\bar{A}) \subset \overline{f(A)}$<br>(iii) For every closed subset B of Y, the set $f^{-1}(B)$ is closed in X. | K3 | CO2 |
| 3 | 18 | (i) State and prove Lebesgue number lemma.<br>(ii) State and prove Uniform continuity theorem. | K4 | CO3 |
| 4 | 19 | Let $X$ be a metrizable space. Prove that the following are equivalent:<br>(i) $X$ is compact.<br>(ii) $X$ is limit point compact.<br>(iii) $X$ is sequentially compact. | K6 | CO4 |
| 5 | 20 | State and prove the Urysohn lemma. | K5 | CO5 |

Z-Z-Z END