22MAP102N

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2025
(First Semester)

Branch - **MATHEMATICS**

REAL ANALYSIS

Time: Three Hours | Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	Let f be defined on $[a,b]$; if f has a local maximum at a point $x \in (a,b)$, and if $f'(x)$ exists, then $f'(x)$ is ______. (a) $= 0$ (b) $\neq 0$ (c) < 0 (d) > 0	K1	CO1
	2	If $f'(x)$ is _____ $x \in (a,b)$, then f is monotonically decreasing. (a) $= 0$ (b) $\neq 0$ (c) ≤ 0 (d) ≥ 0	K2	CO1
2	3	The partition P^* is a refinement of P if ______. (a) $P^* \subseteq P$ (b) $P \subseteq P^*$ (c) $P^* \subset P$ (d) $P^* \supset P$	K1	CO2
	4	If $f \in \mathcal{R}(\alpha)$ on $[a,b]$ and if c is a positive constant, then $f \in \mathcal{R}(c\alpha)$ and $\int_a^b f\, d(c\alpha)$ is ________. (a) $= \int_a^b f\, cd(\alpha)$ (b) $= c\int_a^b f\, d(\alpha)$ (c) $\geq c\int_a^b f\, d(\alpha)$ (d) $\leq c\int_a^b f\, d(\alpha)$	K2	CO2
3	5	The function $f_n(x) = \frac{x^2}{x^2+(1-nx)^2}$ is ________. (a) uniformly bounded (b) uniformly convergent (c) equicontinuous (d) pointwise bounded	K1	CO3
	6	The sequence $\{f_n\}$ is uniformly bounded on E if there exists a number M such that ________, $x \in E, n = 1,2,3, \ldots$. (a) $\lvert f_n(x)\rvert \geq M$ (b) $\lvert f_n(x)\rvert > M$ (c) $\lvert f_n(x)\rvert \leq M$ (d) $\lvert f_n(x)\rvert < M$	K2	CO3
4	7	$\lim_{x\to\infty} x^{-\alpha} \log x =$ ___, for every $\alpha > 0$. (a) ∞ (b) $-\infty$ (c) 0 (d) none	K1	CO4
	8	For $0 < x < \infty, \Gamma(x) =$ ________. (a) $\int_0^\infty t^x e^{-t} dt$ (b) $\int_0^\infty t^{x-1} e^{-t} dt$ (c) $\int_0^\infty t^x e^t dt$ (d) $\int_0^\infty t^{x-1} e^t dt$	K2	CO4
5	9	Let r be a positive integer. If a vector space X is spanned by a set of r vectors, then dim X ______ (a) $\leq r$ (b) $<r$ (c) $\geq r$ (d) $> r$	K1	CO5
	10	The _______ of A is defined to be the dimension of $\mathcal{R}(A)$. (a) basis (b) span (c) range (d) null space	K2	CO5

Cont...

SECTION - B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Suppose f and g are defined on [a, b] and are differentiable at a point $x \in [a, b]$. Then prove that $f + g, fg$ and f/g are differentiable at x.	K2	CO1
		(OR)		
	11.b.	Suppose f is a real differentiable function on $[a, b]$ and suppose $f'(a) < \lambda < f'(b)$. Then prove that there is a point $x \in (a, b)$ such that $f'(x) = \lambda$.		
2	12.a.	If P^* is a refinement of P, then prove that $L(P, f, \alpha) \leq L(P^*, f, \alpha)$ and $U(P^*, f, \alpha) \leq U(P, f, \alpha)$	K2	CO2
		(OR)		
	12.b.	State and prove the fundamental theorem of calculus.		
3	13.a.	Suppose $\{f_n\}$ is a sequence of functions defined on E, and suppose $\lvert f_n(x)\rvert \leq M_n, (x \in E, n = 1,2,3, \ldots\ldots)$. Then prove that $\sum f_n$ converges uniformly on E if $\sum M_n$ converges.	K1	CO3
		(OR)		
	13.b.	If K is a compact metric space, if $f_n \in \mathcal{C}(K)$ for $n = 1, 2, 3, \ldots\ldots$, and if $\{f_n\}$ converges uniformly on K, then prove that $\{f_n\}$ is equicontinuous on K.		
4	14.a.	Suppose the series $\sum_{n=0}^{\infty} c_n x^n$ converges for $\lvert x\rvert < R$, and define $f(x) = \sum_{n=0}^{\infty} c_n x^n$, $\lvert x\rvert < R$. Then show that the series converges uniformly on $[-R + \varepsilon, R - \varepsilon]$, no matter which $\varepsilon > 0$ is chosen. The function f is continuous and differentiable in $(-R, R)$, and $f'(x) = \sum_{n=1}^{\infty} n c_n x^{n-1}, \lvert x\rvert < R$.	K3	CO4
		(OR)		
	14.b.	If $x > 0$ and $y > 0$, then prove that $\beta(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$.		
5	15.a.	Let Ω be the set of all invertible linear operators on R^n, prove that (a). If $A \in \Omega, B \in L(R^n)$, and $\lVert B - A\rVert \cdot \lVert A^{-1}\rVert < 1$, then $B \in \Omega$. (b). Ω is an open subset of $L(R^n)$, and the mapping $A \to A^{-1}$ is continuous on Ω.	K3	CO5
		(OR)		
	15.b.	Suppose **f** maps a convex open set $E \subset R^n$ into R^m, **f** is differentiable in E, and there is a real number M such that $\lVert f'(x)\rVert \leq M$ for every $x \in E$. Then prove that $\lvert \boldsymbol{f}(\boldsymbol{a}) - \boldsymbol{f}(\boldsymbol{b})\rvert \leq M\lvert \boldsymbol{b} - \boldsymbol{a}\rvert$ for all $\boldsymbol{a} \in E$.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	State and prove chain rule for differential.	K2	CO1
2	17	If γ' is continuous on $[a, b]$, then prove that γ is rectifiable, and $\Lambda(\gamma) = \int_a^b \lvert\gamma'(t)\rvert dt$.	K3	CO2
3	18	State and prove the Stone-Weierstrass theorem.	K1	CO3
4	19	State and prove Taylor's theorem.	K4	CO4
5	20	State and the inverse function theorem.	K4	CO5

Z-Z-Z END