PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2025
(Second Semester)

Branch - **MATHEMATICS**

PARTIAL DIFFERENTIAL EQUATIONS

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 × 1 = 10)

Question No.	Question	K Level	CO
1	The two equations $f(x,y,z,p,q)=0$ and $g(x,y,z,p,q)=0$ is said to be compatible if a) $[f,g]=0$ b) $[f,g]\neq 0$ c) $[f,g]=1$ d) $[f,g]\neq 1$	K1	CO1
2	Which of the following is an charpit's equation? a) $\frac{dx}{f_p}=\frac{dy}{f_q}=\frac{dz}{(pf_p+qf_q)}=\frac{dp}{-(f_x+pf_z)}=\frac{dq}{(f_y+qf_z)}$ b) $\frac{dx}{f_p}=\frac{dy}{f_q}=\frac{dz}{(pf_p+qf_q)}=\frac{dp}{-(f_x+pf_z)}=\frac{dq}{-(f_y+qf_z)}$ c) $\frac{dx}{f_p}=\frac{dy}{f_q}=\frac{dz}{(pf_p+qf_q)}=\frac{dp}{(f_x+pf_z)}=\frac{dq}{(f_y+qf_z)}$ d) $\frac{dx}{f_p}=\frac{dy}{f_q}=\frac{dz}{-(pf_p+qf_q)}=\frac{dp}{-(f_x+pf_z)}=\frac{dq}{-(f_y+qf_z)}$	K2	CO1
3	Choose the correct telegraphy equation a) $\frac{\partial^2\varphi}{\partial x^2}=k\frac{\partial\varphi}{\partial t}$ b) $\frac{\partial^2\varphi}{\partial x^2}=\frac{1}{k}\frac{\partial\varphi}{\partial t}$ c) $\frac{\partial^2\varphi}{\partial x^2}=-k\frac{\partial\varphi}{\partial t}$ d) $\frac{\partial^2\varphi}{\partial x^2}=\frac{-1}{k}\frac{\partial\varphi}{\partial t}$	K1	CO2
4	Canonical form of the equation $z_{xx}+2z_{xy}+z_{yy}=0$ is a) $G_{\eta\eta}=-1$ b) $G_{\eta\eta}=1$ c) $G_{\eta\eta}=0$ d) $G_{\eta\eta}=2$	K2	CO2
5	The equation $\nabla^2\psi=0$ is known as a) Diffusion equation b) Wave equation c) Heat equation d) Laplace equation	K1	CO3
6	In the three dimensional case, the solution of the exterior Dirichlet problem is unique then a) $\psi(x,y,z)<\frac{c}{r}$ b) $\lvert\psi(x,y,z)\rvert\leq\frac{c}{r}$ c) $\lvert\psi(x,y,z)\rvert>\frac{c}{r}$ d) $\lvert\psi(x,y,z)\rvert\geq\frac{c}{r}$	K2	CO3
7	Choose the correct d 'Alembert's solution of the one dimensional wave equation. a) $y=\frac{1}{2}\{\eta(x+ct)+\eta(x-ct)\}+\frac{1}{2c}\int_{x-ct}^{x+ct}v(\xi)\,d\xi$ b) $y=\frac{1}{2}\{\eta(x+ct)-\eta(x-ct)\}+\frac{1}{2c}\int_{x-ct}^{x+ct}v(\xi)d\xi$ c) $y=\frac{1}{2}\{\eta(x+ct)-\eta(x-ct)\}-\frac{1}{2c}\int_{x-ct}^{x+ct}v(\xi)d\xi$ d) $y=\frac{1}{2}\{\eta(x+ct)+\eta(x-ct)\}-\frac{1}{2c}\int_{x-ct}^{x+ct}v(\xi)\,d\xi$	K1	CO4
8	Wave equation is one of the classification of a) Eliptic Equation b) Hyperbolic Equation c) Parabolic Equation d) None of the above	K2	CO4
9	The one-dimensional diffusion equation is a) $\frac{\partial^2\theta}{\partial x^2}=k\frac{\partial\theta}{\partial t}$ b) $\frac{\partial^2\theta}{\partial x^2}=-k\frac{\partial\theta}{\partial t}$ c) $\frac{\partial^2\theta}{\partial x^2}=\frac{1}{k}\frac{\partial\theta}{\partial t}$ d) $\frac{\partial^2\theta}{\partial x^2}=\frac{-1}{k}\frac{\partial\theta}{\partial t}$	K1	CO5
10	The deterministic form of diffusion equation is a) <0 b) >0 c) $=0$ d) None of the above	K2	CO5

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SECTION - B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Question No.	Question	K Level	CO
11.a.	Show that the equation $xp = yq, z(xp = yq) = 2xy$ are compatiable and hence solve them.	K2	CO1
	(OR)		
11.b.	Find the general integral of the linear partial differential equation $z(xp - yq) = y^2 - x^2$.		
12.a.	If the operator $F(D, D')$ is irreducible. Then prove that the order in which the lineal factors occur is unimportant.	K3	CO2
	(OR)		
12.b.	Find a particular integral of the equation $\left(D^{2}- D'\right)z = 2y - x^2$.		
13.a.	Prove that $\nabla^2 \psi = 0$.	K3	CO3
	(OR)		
13.b.	Prove that the solution of a certain Neumann Problem can differ from one another by a constant only.		
14.a.	Find approximate values for the first three eigenvalues of a square nembuance of side 2.	K4	CO4
	(OR)		
14.b.	Derive the Riemann-Volterra solution of the one-dimensional wave equation.		
15.a.	Find the temperature in a sphere of radius 'a' when its surface is maintained at zero temperature and its initial is $f(r, \theta)$.	K5	CO5
	(OR)		
15.b.	Determine the function $\theta\ (r, t)$ satisfying $\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} = \frac{1}{k}\frac{\partial\theta}{\partial t}$ $t > 0, 0 < r < a$ and the conditions $\theta(r, 0) = 0, \theta(a, t) = f(t)$.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Question No.	Question	K Level	CO
16	Verify that $z = ax + by + a + b - ab$ is a complete integral of the partial differential equation $z = px + qy + p + q - pq$, where a, b are arbitrary constants. Show that the envelope of all planes corresponding to complete integrals provides a singular solution of the differential equation and determine a general solution by finding the envelope of those planes that pass through the origin.	K5	CO1
17	Reduce the equation $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form.	K2	CO2
18	Show that the surfaces $(x^2 + y^2)^2 - 2a^2\ (x^2 - y^2) + a^4 = c$ can form of equipotential surfaces and find the general form of the corresponding potential function.	K3	CO3
19	A thin membrance of great extent is released from rest in the position $z = f(x, y)$. Determine the displacement at any subsequent time.	K3	CO4
20	State and prove Duhamel's theorem.	K4	CO5

Z-Z-Z END