TOTAL PAGES: 2
22MAP209N

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2025
(Second Semester)

Branch- **MATHEMATICS**

MEASURE THEORY

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	$m^*(\emptyset) =$ ______ i) ∞ ii) 0 iii) $\emptyset$ iv) 1	K1	CO1
	2	For any measurable function f, $ess\ sup\ f$ i) $< sup\ f$ ii) $\leq sup\ f$ iii) $> sup\ f$ iv) $\geq sup\ f$	K2	
2	3	$\int_1^\infty dx/x =$ ______ i) ∞ ii) 0 iii) $\emptyset$ iv) 1	K1	CO2
	4	f is an integrable function, then $\lvert\int f dx\rvert =$ ______ i) $\leq \int f dx$ ii) $\leq f \int dx$ iii) $\leq \int \lvert f\rvert dx$ iv) $\leq \int dx$	K2	
3	5	A ring is called a ____ if it is closed under the formation of countable unions. i) σ-field ii) integral domain iii) group iv) σ-ring	K1	CO3
	6	The completion of a σ-finite measure is ______ i) finite ii) infinite iii) σ-finite iv) σ-infnite	K2	
4	7	If $\mu(X) < \infty$ and $0 < p < q \leq \infty$ then, i) $L^q(\mu) \subseteq L^p(\mu)$ ii) $L^q(\mu) > L^p(\mu)$ iii) $L^q(\mu) \geq L^p(\mu)$ iv) $L^q(\mu) \supseteq L^p(\mu)$	K1	CO4
	8	Every function convex on an open interval is ____ i) discontinuous ii) non-convex iii) continuous iv) none of the above	K2	
5	9	If A is a positive set with respect to v, if for $E \in \mathcal{S}$, ______ then μ is a measure. i) $\mu(E) = v(A)$ ii) $\mu(E) \neq v(A)$ iii) $\mu(E) \neq v(E \cap A)$ iv) $\mu(E) = v(E \cap A)$	K2	CO5
	10	If μ, v are signed measures on $[\![X, \mathcal{S}]\!]$ and $v(E) = 0$ whenever____, then v is absolutely continuous with respect to $\mu, v \ll \mu$ i) $\lvert\mu\rvert(E) = 0$ ii) $\lvert\mu\rvert(E) = \infty$ iii) $\lvert\mu\rvert(E) = 1$ iv) $\lvert\mu\rvert(E) \neq 0$	K1	

Cont...

SECTION - B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	Show that, for any set A and any $\epsilon > 0$ there is an open set O containing A and such that $m^*(O) \leq m^*(A) + \epsilon$.	K2	CO1
		(OR)		
	11.b.	Let c be any real number and let f and g be real-valued measurable functions defined on the same measurable set E. Then show that $f + c, cf, f + g, f - g$ and fg are measurable.		
2	12.a.	Let f and g be non-negative measurable functions. Then, examine $\int f\,dx + \int g\,dx = \int (f + g)\,dx$.	K4	CO2
		(OR)		
	12.b.	If f is Riemann integrable and bounded over the finite interval $[a, b]$, then examine f is integrable and $R\int_a^b f\,dx = \int_a^b f\,dx$.		
3	13.a.	Let μ^* be the outer measure on $\mathcal{H}(\mathcal{R})$ defined by μ on $\mathcal{R}$, then inspect $\mathcal{S}^*$ contains $\mathcal{S}(\mathcal{R})$, the σ-ring generated by $\mathcal{R}$.	K4	CO3
		(OR)		
	13.b.	Inspect: The completion of a σ-finite measure is σ-finite .		
4	14.a.	Build: Let ψ be convex on (a, b) and $a < s < t < u < b$ then $\psi(s,t) \leq \psi(s,u) \leq \psi(t,u)$. If ψ is strictly convex, then equality will not occur.	K3	CO4
		(OR)		
	14.b.	State and Prove Holder's inequality.		
5	15.a.	Summarize: Let v be a signed measure on $[\![X, \mathcal{S}]\!]$. Then there exist measures v^+ and v^- on $[\![X, \mathcal{S}]\!]$ such that $v = v^+ - v^-$ and $v^+ \perp v^-$. The measures v^+ and v^- are uniquely defined by v and $v = v^+ - v^-$ is said to be the Jordan decomposition of v.	K2	CO5
		(OR)		
	15.b.	Give the outline for A countable union of sets positive with respect to a signed measure v is a positive set.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	Explain: The class $\mathcal{M}$ is a σ-algebra.	K5	CO1
2	17	State and prove Fatou's Lemma.	K5	CO2
3	18	If μ is a σ-finite measure on a ring $\mathcal{R}$, then inspect it has a unique extension to the σ-ring $\mathcal{S}(\mathcal{R})$.	K4	CO3
4	19	State and prove Minkowski's inequality.	K5	CO4
5	20	State and Prove Radon-Nikodym Theorem.	K5	CO5

Z-Z-Z END