22MAP101N

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

MSc DEGREE EXAMINATION MAY 2025
(First Semester)

Branch - **MATHEMATICS**

ALGEBRA

Time: Three Hours | Maximum: 75 Marks

SECTION-A (10 Marks)
Answer ALL questions
ALL questions carry EQUAL marks (10 × 1 = 10)

Module No.	Question No.	Question	K Level	CO
1	1	Any subgroup of order p^{n-1} in a group G of order p^n where p is a prime number is (a) Normal in G (b) Finite in G (c) Isomorphic in G (d) Cyclic in G	K1	CO1
	2	If $p^m/o(G)$, and $p^{m+1} \nmid o(G)$ then G has a subgroup of order (a) p^{m+1} (b) p^n (c) p^m (d) p^{m-1}	K2	CO1
2	3	A polynomial is said to be integer monic if all its coefficients are integers and its highest coefficient is (a) 0 (b) 1 (c) n (d) 2	K1	CO2
	4	If $a \epsilon R$ is an irreducible element and a/bc then (a) $a\|b$ and $a\|c$ (b) $a\|b$ or $a\|c$ (c) $a \nmid b$ and $a \nmid c$ (d) $ab\|c$	K2	CO2
3	5	If $a \in K$ is algebraic of degree n over F then $[F(a):F] =$ (a) n^2 (b) $n+1$ (c) n (d) $n-1$	K1	CO3
	6	If $a \in K$ is algebraic of degree n over F then $[F(a):F] =$ (a) n^2 (b) $n+1$ (c) n (d) $n-1$	K2	CO3
4	7	For some $\alpha \epsilon K$ the extension K of F is a simple extension of F if (a) $K = F(\alpha)$ (b) $F = K(\alpha)$ (c) $K = F$ (d) $K^2 = \alpha$	K1	CO4
	8	Any finite extension of a field of characteristic zero is a (a) Finite extension (b) Simple extension (c) Infinite extension (d) Multiple extension	K2	CO4
5	9	$T \in A(V)$ is unitary if and only if (a) $TT^* = 0$ (b) $TT^{-1} = 0$ (c) $TT^* = 1$ (d) $T = T^*$	K1	CO5
	10	If $T \in A(V)$ is called Hermitian if (a) $T^* = T^{-1}$ (b) $T^* \neq T^{-1}$ (c) $T^* = T^2$ (d) $T^* = T$	K2	CO5

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SECTION - B (35 Marks)

Answer ALL questions

ALL questions carry EQUAL Marks (5 × 7 = 35)

Module No.	Question No.	Question	K Level	CO
1	11.a.	If ϕ is a homomorphism of G in to $\bar{G}$ with kernel K then prove that K is a normal subgroup of G.	K3	CO1
		(OR)		
	11.b.	If G is a group then prove that $\mathcal{A}(G)$, the set of automorphisms of G is also a group.		
2	12.a.	If $f(x)$ *and* $g(x)$ are two primitive polynomials then prove that $f(x)g(x)$ is a primitive polynomial.	K4	CO2
		(OR)		
	12.b.	If $f(x)$ in $R[x]$ is both primitive and irreducible as an element of $R[x]$, then prove that it is irreducible as an element of $F[x]$? Prove the converse also.		
3	13.a.	Let $f(x) \in F[x]$ be of degree $n \geq 1$then prove that there is an extension E of F of degree at most $n!$ in which $f(x)$ has n roots.	K3	CO3
		(OR)		
	13.b.	If $p(x) \in F[X]$ and if K is an extension of F then prove that for any element $b \in K, p(x) = (x - b)q(x) + p(b)$ where $q(x) \in K[x]$ and where $deg\, q(x) = deg p(x) - 1$.		
4	14.a.	For any $f(x), g(x) \in F[x]$ and any $\alpha \in F$ then prove that (i). $(f(x) + g(x))' = f'(x) + g'(x)$ (ii). $(\alpha f(x)' = \alpha f'(x)$ (iii). $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$	K5	CO4
		(OR)		
	14.b.	Prove that the fixed field of G is a subfield of K.		
5	15.a.	If $\lambda \in F$ is a characteristic root of $T \in A(V)$ then prove that λ is a root of the minimal polynomial of T.	K4	CO5
		(OR)		
	15.b.	If V is n-dimensional over F and if $T \in A(V)$ has all its characteristic roots in F then Prove that T satisfies a polynomial of degree n over F.		

SECTION -C (30 Marks)

Answer ANY THREE questions

ALL questions carry EQUAL Marks (3 × 10 = 30)

Module No.	Question No.	Question	K Level	CO
1	16	State and prove Cauchy's theorem for abelian groups.	K5	CO1
2	17	Prove that the ideal $A = (p(x))$in $F[x]$ is maximal ideal if and only if $p(x)$ is irreducible over F.	K4	CO2
3	18	If L is a finite extension of K and if K is a finite extension of F then prove that L is a finite extension of F.	K4	CO3
4	19	If F is of characteristic 0 and if a, b are algebraic over F then prove that there exists an element $c \in F(a, b)$ such that $(a, b) = F(c)$.	K5	CO4
5	20	If $T \in A(V)$ has all its characteristic roots in F then prove that there is a basis of V in which the matrix of T is triangular.	K4	CO5

Z-Z-Z END