PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2025
(Sixth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

COMPLEX ANALYSIS

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1 Choose the correct statement from the following.
a) A continuous function is differentiable
b) A differentiable function is continuous
c) Every single-valued function is continuous
d) Every single-valued function is differentiable

2 Contour is a type of _________ in the complex plane.
a) curve b) area c) closed region d) closed curve

3 $\int_C \frac{e^z dz}{z} =$ ________, where C is the circle $|z|=1$
a) $2\pi i$ b) $\pi i/2$ c) $4\pi i$ d) $\frac{\pi}{2}i$

4 The singular point of $f(z)=\frac{1}{z}$ *is*
a) 0 b) 1 c) i d) $-i$

5 The residue of cotz at the simple pole z=0 is
a) 0 b) ∞ c) z d) 1

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a If a function $f(z)=u(x,y)+iv(x,y)$ is analytic function in a domain then prove that its component functions u and v are harmonic in D.

OR

b Suppose that a function $f(z)=u(x,y)+iv(x,y)$ and its conjugate $\overline{f(z)}=u(x,y)-iv(x,y)$ are both analytic in a domain D show that $f(z)$ must be constant in the domain.

7 a Let C denote a contour of length L, and suppose that a function f (z) is piecewise continuous on C. If M is a nonnegative constant such that $|f(z)| \le M$ for all points z on C at which f (z) is defined, then prove that $\left|\int_C f(z)dz\right| \le ML$.

OR

b Suppose $f(z)$ is an analytic function in a simply-connected region D, $A(a)$ and $B(b)$ are two points in D. Let C be any arbitrary chosen simple rectifiable arc in D oriented from A to B then prove that the integral $\int_C f(z)dz$ does not depend on C but depends on a and b.

Cont...

8 a State and prove Liouville's e theorem.

OR

b Let f(z) be a function is analytic inside and on a simple closed curve C. Let z_0 be any point in the interior of C. then prove that $f(z_0)=\frac{1}{2\pi i}\int_C \frac{f(z)}{z-z_0}$.

9 a Let f(z) be a function which is analytic inside and on a simple closed curve C except for a finite number of singular points $z_1, z_2, \ldots\ldots\ldots\ldots z_n$ inside C. Then prove that $\int_C f(z)\,dz=\frac{1}{2\pi i}\sum_{j=1}^{n}\text{Res}\{f(z;z_j\}$

OR

b Find the circle C, where $\int_C \frac{1}{z^2-4}dz=0$.

10 a If f(z) and g(z) are analytic inside and on a simple closed curve C and if $|g(z)|<|f(z)|$ on C then prove that f(z)+g(z) and f(z) have the same number of zeroes inside C.

OR

b Prove that every polynomial of degree ≥1 has at least one zero.

SECTION -C (30 Marks)

Answer ALL questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a Derive Cauchy's –Riemann equation in polar form.

OR

b (i) Prove that the real and imaginary parts of an analytic function are harmonic functions.

(ii) Calculate the analytic function f(z)= u+iv of which the real part is $u=e^x(x\cos y-y\sin y)$

12 a State and prove Cauchy-Goursat theorem.

OR

b Calculate the residue of $\frac{1}{(z^2+a^2)^2}$ at z= ai.

13 a State and prove Laurent's theorem.

OR

b State and prove maximum modulus theorem.

14 a State and prove Cauchy's Residue theorem.

OR

b Evaluate the following integrals.

(i) $\int_C \frac{1}{2z+1}dz$; C is $|z|=1$

(ii) $\int_C \frac{1}{2z+3}dz$; C is $|z|=2$

15 a Evaluate $\int_{-\infty}^{\infty}\frac{\cos x}{(x^2+a^2)(x^2+b^2)}dx \quad (a>b>0)$

OR

b Evaluate the poles of $f(z)=\frac{z^2+4}{z^3+2z^2+2z}$ and determine the residue at the poles.

Z-Z-Z

END