23MCU520

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2025
(Fifth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

REAL ANALYSIS

Time: Three Hours　　　　　　　　Maximum: 50 Marks

SECTION-A (5 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks　　　　(5 x 1 = 5)

1. In a metric space $< M, \rho >$, for all $x, y, z \in M$, the triangle inequality is ____
   (i) $\rho(x,y) \leq \rho(x,z) + \rho(z,y)$ (ii) $\rho(x,y) \geq \rho(x,z) + \rho(z,y)$
   (iii) $\rho(x,y) < \rho(x,z) + \rho(z,y)$ (iv) $\rho(x,y) > \rho(x,z) + \rho(z,y)$

2. Let G be an open subset of a metric space M, then $G' = M - G$ is ____
   (i) open　(ii) closed
   (iii) ϕ　(iv) M

3. Which among the following is a connected subset of R^1?
   (i) $[1,3] \cup [3,5]$　(ii) $(0, \infty)$
   (iii) $(4,6] \cup [7,8)$　(iv) $[0,2] \cup [3,4]$

4. Identify the compact metric space from the following
   (i) R^1　(ii) (0,1)
   (iii) [a,b]　(iv) Infinite subset of R^d

5. Which among the following is not a correct statement?
   (i) If $J = (2,5)$, then $|J| = 7$　(ii) $\left|\int_a^b f\right| \leq \int_a^b |f|$
   (iii) $\int_a^b f + g = \int_a^b f + \int_a^b g$　(iv) if $f \geq 0$, then $\int_a^b f \geq 0$.

SECTION - B (15 Marks)

Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks　　　　(5 x 3 = 15)

6 a Show that the set of real numbers $\mathbb{R}$ is a metric space with the metric $\rho(x,y) = |x - y|$.

OR

b Let $< M, \rho >$ be a metric space. If $\{s_n\}_{n=1}^{\infty}$ is a convergent sequence of points in M, then prove that $\{s_n\}_{n=1}^{\infty}$ is Cauchy.

7 a If F_1 and F_2 are closed subsets of the metric space M, then $F_1 \cup F_2$ is closed.

OR

B If f and g are real valued functions, if f is continuous at a and if g is continuous at $f(a)$, then $g \circ f$ is continuous at $x = a$.

8 a If $u = \{u_n\}_{n=1}^{\infty} \in l^2$. Let $Tu = \left\{\frac{u_n}{2}\right\}_{n=1}^{\infty}$. Prove that T is a contraction on l^2.

OR

b If the subset A of the metric space $< M, \rho >$ is totally bounded, then A is bounded.

9 a Let f be a continuous function from the compact metric space M_1 into the metric space M_2. Then prove that the range $f(M_1)$ of f is also compact.

OR

b If f is one to one continuous function from the compact metric space M_1 into the metric space M_2, then prove that f^{-1} is continuous on M_2 and hence f is a homeomorphism of M_1 onto M_2.

10 a If $f, g \in \mathcal{R}[a,b]$, then prove that $f+g \in \mathcal{R}[a,b]$ and

$$\int_a^b f+g = \int_a^b f + \int_a^b g.$$

OR

b State and prove Rolle's theorem.

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a Let $< M, \rho >$ be a metric space and let 'a' be a point in M. Let f and g be real valued functions whose domains are subsets of M. If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = N$, then

(i) $\lim_{x\to a}[f(x) + g(x)] = L + N$

(ii) $\lim_{x\to a}[f(x) * g(x)] = LN$.

OR

b (i) Prove that $\lim_{x\to 0} \sin\left(\frac{1}{x}\right) \neq L$.

(ii) Prove that $\lim_{x\to\infty}\left(\frac{1}{x^2}\right) = 0$.

12 a Prove that "The set R^1 is of the second category".

OR

b (i) Show that the function $f(x) = x^2 + 2x$ is continuous at $x = 3$.

(ii) If E is any subset of a metric space M, then $\bar{E} = \bar{\bar{E}}$.

13 a Prove that "The subset A of R^1 is said to be connected if and only if whenever $a \in A, b \in B$, with $a < b$, then $c \in A$ for any c such that $a < c < b$. That is, whenever $a \in A, b \in B, a < b$, then $(a,b) \in A$."

OR

b Let $< M, \rho >$ be a complete metric space. If T is a contraction on M, then there is one and only one point $x \in M$ such that $Tx = x$. That is T has precisely one fixed point.

14 a Prove that "If M is a compact metric space, then M has the Heine Borel property."

OR

b Let $< M_1, \rho_1 >$ be a metric space and let A be a dense subset of M_1. If f is a uniformly continuous function from $< A, \rho_1 >$ into a complete metric space $< M_2, \rho_2 >$, then prove that f can be extended to a uniformly continuous function F from M_1 into M_2.

15 a Suppose g has a derivative at c and that f has a derivative at $g(c)$. Then prove that $\phi = fog$ has a derivative at c and $\phi'(c) = f'(g(c))g'(c)$.

OR

b If f is continuous on the closed bounded interval [a,b] and if

$$F(x) = \int_a^x f(t)dt, \quad a \leq x \leq b,$$

Then prove that $F'(x) = f(x), \ a \leq x \leq b$.

Z-Z-Z END