PSG COLLEGE OF ARTS & SCIENCE (AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2025

(Third Semester)

Branch - MATHEMATICS WITH COMPUTER APPLICATIONS

ADVANCED MATHEMATICAL STATISTICS - I

Maximum: 75 Marks Time: Three Hours

SECTION-A (10 Marks)

Answer ALL questions

ALL questions carry EQUAL marks

 $(10 \times 1 = 10)$

Module No.	Question No.	Question	K Level	СО
1	1	If A and B are two independent events, then $P(A \cap \overline{B})$ is a) $P(A) + P(B)$ b) $P(A)P(B)$ c) $P(A) - P(B)$ d) $\frac{P(A)}{P(B)}$	K1	CO1
	2	If $P(A) = \frac{7}{11}$, $P(B) = \frac{6}{11}$ and $P(AUB) = \frac{8}{11}$, then $P(A B)$ is a) $\frac{6}{8}$ b) $\frac{6}{7}$ c) $\frac{5}{6}$ d) $\frac{1}{2}$	K2	CO1
2	3	For two random variables X and Y , if the joint probability mass function is $P(X = x, Y = y)$, the marginal probability $P(X = x)$ is given by: a) $\sum_{x} P(X = x, Y = y)$ b) $\sum_{y} P(X = x, Y = y)$ c) $\int P(X = x, Y = y) dy$ d) $P(X = x, Y = y)/P(Y = y)$	K1	CO2
	4	The cumulative distribution function $F(x)$ for a continuous random variable X represents: a) The probability that X is greater than x . b) The probability that X is equal to X . c) The probability that X is less than or equal to X . d) The probability that X is less than X .	K2	CO2
	5	If $Var(X) = 1$, then $Var(2X + 3)$ is a)5 b) 13 c) 9 d) 4	K1	CO3
3	6	If $E(X^r)$ exists then $E(X^s)$ also exist for all a) $1 \le s \le r$ b) $1 < r < s$ c) $1 < s < r$ d) $-1 \le s \le 1$	K2	CO3
4	7	If X is a random variable, $E(e^{itx})$ is known as a) characteristic function b) moment generating function c) probability generating function d) the x^{th} moment	K1	CO4
	8	In M.G.F standard variance is a) -1 b) 0 c) ∞ d) 1	K2	CO4
5	9	does not have additive or reproductive property. a) Poisson distribution b) Binomial distribution c) Poisson variate d) Binomial variate	K1	CO5
	10	The moment-generating function of normal distribution is a) $e^{\mu t - t^2/2}$ b) $e^{\mu t - t^2\sigma^2/2}$ c) $e^{\mu t + t^2/2}$ d) $e^{\mu t + t^2\sigma^2/2}$	K2	CO5

SECTION - B (35 Marks) Answer ALL questions

LL questions carr	y EQUAL Marks	$(5 \times 7 = 3)$	35)
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Module No.	Question No.	Question	K Level	CO	
1	11.a.	State and prove Multiplication theorem on probability			
		(OR)			
	11.b.	Sixty percent of the employees of the XYZ corporation are college graduates. Of these, ten percent are in sales. Of the employees who did not graduate from college, eighty percent are in sales. What is the probability that (i) an employee selected at random is in sales? (ii) an employee selected at random is neither in sales nor a college graduate?	K5/ K4	CO1	

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2	12.a.	When random variable is said to be discrete and obtain any two properties of distribution function.		,
	(OR)			
	12.b.	The joint probability distribution of two random variables X and Y is given by: $P(X = 0, Y = 1) = \frac{1}{3}$, $P(X = 1, Y = -1) = \frac{1}{3}$, and	K2/ K4	CO2
		$P(X = 1, Y = 1) = \frac{1}{3}$. Find (i) marginal distribution of X and Y, and		
		(ii) the conditional distribution of X given $Y = 1$.		
3	13.a.	 (a) State and prove multiplicative property of the mathematical expectation. (b) If X and Y are two random variables such that X ≤ Y, prove that E(X) ≤ E(Y) 	K3/ K5	CO3
-	(OR)			
	13.b.	State and Prove linearity property of variance.		
	14.a.	Define characteristic function of random variable. Prove that the characteristic function of the sum of two independent variable is equal to the product of their characteristic function.	K5/	CO4
4	(OR)		K2	004
	14.b.	What is moment generating function? Also find the <i>m.g.f.</i> of the random variable whose moments are: $\mu'_r = (r+1)! 2^r$.		
5	15.a.	A department in a works has 10 machines which may need adjustment from time to time during the day. Three of these machines are old, each having a probability of 1/11 of needing adjustment during the day, and 7 are new, having corresponding probabilities of 1/21. Assuming that no machines need adjustment twice on the same day. Determine the probabilities that on particular day (i) just 2 old and no new machines need adjustment. (ii) if just 2 machines need adjustment, they are of the same type.	K4/ K2	CO5
	15.b.	Mention the chief characteristic of the normal distribution and normal probability curve.		

SECTION -C (30 Marks) Answer ANY THREE questions

 $(3\times10=30)$ ALL questions carry EQUAL Marks \mathbf{K} Module Question CO Question Level No. No. CO₁ K5 State and Prove Baye's theorem. 16 1 The diameter, say X, of an electric cable, is assumed to a be a continuous random variable with p.d.f: $f(x) = 6x(1-x), 0 \le x \le 1$. (i) Check that the above is a p.d.f. K4 CO₂ 17 2 (ii) Obtain the expression for the c.d.f. of X. (iii) Compute $P\left(X \le \frac{1}{2} | \frac{1}{3} \le X \le \frac{2}{3}\right)$, and (iv) Determine the number k such that P(X < k) = P(X > k). Two random variables X and Y have the following joint probability density function $f(x,y) = \begin{cases} 2-x-y; 0 \le x \le 1, 0 \le y \le 1 \\ 0, \text{ otherwise} \end{cases}$ K4 CO3 18 3 Find (i) conditional density functions, (ii) Var(X) and Var (Y); (iii) Covariance between X and Y. Obtain the properties of moment generating function. K2 CO₄ 19 4 Derive moments of Poisson distribution and hence find its mean, K5 CO₅ 20 5 variance, coefficient of skewness and kurtosis.