24MCU310 / 23MCU309

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2025
(Third Semester)

Branch - **MATHEMATICS WITH COMPUTER APPLICATIONS**

PARTIAL DIFFERENTIAL EQUATIONS AND FOURIER SERIES

Time: Three Hours Maximum: 75 Marks

SECTION-A (10 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** marks (10 × 1 = 10)

Question No.	Question	K Level	CO
1	An equation involving partial derivative coefficients of a function of two or more variables is _____ equation. (a) partial differential (b) ordinary differential (c) fractional differential (d) differential equation	K1	CO1
2	The _____ of a partial differential equation is the highest order of any derivative present in the equation. (a) form (b) order (c) degree (d) linear form	K1	CO1
3	How many arbitrary constants are generally present in the general solution of a first-order PDE? (a) Zero (b) One (c) Two (d) Four	K1	CO2
4	A complete integral of a first-order PDE is _____. (a) a solution that satisfies the PDE for all values of constants (b) a specific solution obtained by assigning specific values to constants (c) a solution that contains all possible solutions (d) a solution that is not defined	K2	CO2
5	A second-order PDE is characterized by _____. (a) the presence of only first derivatives (b) the presence of second derivatives (c) the absence of derivatives (d) the presence of third derivatives	K2	CO3
6	An equation is said to be of order two if it involves _____ one of the differential coefficients $\frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial x \partial y}, \frac{\partial^2 z}{\partial y^2}$. (a) atleast (b) atmost (c) exactly (d) no	K2	CO3
7	f(x) is an _____ function if $f(-x) = -f(x)$. (a) even (b) odd (c) cosine (d) sine	K1	CO4
8	The coefficients a_n in a Fourier series are associated with _____ function. (a) sine (b) cosine (c) exponential (d) normal	K2	CO4
9	The one-dimensional heat equation is given by _____. (a) $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ (b) $\frac{\partial u}{\partial x} = c^2 \frac{\partial^2 u}{\partial x^2}$ (c) $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial t^2}$ (d) $\frac{\partial u}{\partial t} = c \frac{\partial^2 u}{\partial x^2}$	K2	CO5
10	$\Delta^2 u = 0$ is known as _____ equation. (a) dirichlet (b) laplace (c) normal (d) wave	K1	CO5

SECTION - B (35 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 × 7 = 35)

Question No.	Question	K Level	CO
11.a.	Eliminate a and b from $z = axe^y + \left(\frac{1}{2}\right) a^2 e^y + b$.	K2	CO1
	(OR)		
11.b.	Eliminate the arbitrary functions f and F from $y = f(x - at) + F(x + at)$.		

Cont...

12.a.	Show that the equations $xp - yq = x$ and $x^2p + q = xz$ are compatible and find their solution.	K3	CO2
	(OR)		
12.b.	Find a complete integral of $zpq = p + q$.		
13.a.	Solve: $ar = xy$.	K3	CO3
	(OR)		
13.b.	Solve: $xr = p$.		
14.a.	A sinusoidal voltage Esin ωt, where t is time, is passed through a half-wave rectifier that clips the negative portion of the wave. Find the Fourier series of the resulting periodic function $u(t) = \begin{cases} 0 & if \ -L < t < 0 \\ Esin\,\omega t & if \ \ 0 < x < L \end{cases}$ $p = 2L = \frac{2\pi}{\omega}$, $L = \frac{\pi}{\omega}$.	K3	CO4
	(OR)		
14.b.	Find the Fourier series of the function $f(x)$ if $f(x) = \begin{cases} -k & if \ -2 < x < 0 \\ k & if \ \ 0 < x < 2 \end{cases}$ $p = 2L = 4,\ L = 2.$		
15.a.	Find the temperature in a laterally insulated bar of length L whose ends are kept at temperature 0, assuming that the initial temperature is $f(x) = \begin{cases} x & if \ \ 0 < x < L/2 \\ L - x & if \ \ L/2 < x < L \end{cases}$	K3	CO5
	(OR)		
15.b.	Find the temperature $u(x,t)$ in a laterally insulated copper bar 80 cm long if the initial temperature is $100\ sin\ (\pi x/80)^0$C and the ends are kept at 0^0C. How long will it take for the maximum temperature in the bar to drop to 50^0C ? Physical data for copper: density 8.92 g/cm^3, specific heat 0.092 $cal/(g^oC)$, thermal conductivity 0.95 $cal/(cm\ sec^oC)$.		

SECTION -C (30 Marks)

Answer **ANY THREE** questions

ALL questions carry **EQUAL** Marks (3 × 10 = 30)

Question No.	Question	K Level	CO
16	Form a partial differential equation by eliminating the function ϕ from $\phi(x^2 + y^2 + z^2, z^2 - 2xy) = 0$.	K2	CO1
17	Find a complete integral of $yzp^2 - q = 0$.	K3	CO2
18	Solve: $xys - qy = x^2$.	K3	CO3
19	Find the two half-range expansions of the function $f(x) = \begin{cases} \frac{2k}{L}x & if\ 0 < x < \frac{L}{2} \\ \frac{2k}{L}(L - x) & if \ \ \frac{L}{2} < x < L \end{cases}$	K3	CO4
20	Prove the one-dimensional wave equation.	K3	CO5

Z-Z-Z END