22MCU102N

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2025
(First Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

ORDINARY DIFFERENTIAL EQUATIONS AND LAPLACE TRANSFORMS

Time: Three Hours | Maximum: 75 Marks

SECTION-A (10 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 × 1 = 10)

1. Write the differential equation that the rate of change of the volume V of water in a drinking tank is proportional to the square root of the depth of water in the tank?
   a) $\frac{dv}{dt} = ky$ b) $\frac{dv}{dt} = k\sqrt{y}$ c) $\frac{dv}{dt} = -k\sqrt{y}$ d) $\frac{dv}{dt} = \frac{k}{y}$

2. Solve $\frac{dy}{dx} = 2xy$
   a) $x = y^2 + c$ b) $y = x + c$ c) $y = 2x + c$ d) $y = x^2 + c$

3. Suppose $y_1, y_2, \dots, y_n$ are n solutions of the homogeneous n^{th} order equation which are linearly independent then what about wronskian?
   a) $w = 0$ b) $w \neq 0$ c) $w = \infty$ d) None

4. Write the general solution of homogeneous equations $y'' + p(x)y' + q(x)y = 0$
   a) $y(x) = c_1y_1(x) + c_2y_2(x)$ b) $y(x) = (c_1x + c_2)e^x$
   c) $y(x) = c_1 \cos x + c_2 \sin x$ d) None

5. If the characteristic equation $a_nr^n + a_{n-1}r^{n-1} + \dots + a_1r + a_0 = 0$ has unrepeated pair of complex conjugate roots $a \pm ib$ write down the general solutions?
   a) $e^{ax}(\cos bx - i\sin bx)$ b) $e^{ax}(c_1 \cos bx + c_2 \sin bx)$
   c) $(c_1 + c_2)e^{ax} \cos bx + i(c_1 - c_2)e^{ax} \sin bx$ d) None

6. Find the general solution of $a_0y^{(n)} + a_1y^{(n-1)} + \dots + a_0y = f(x)$
   a) $y = y_c + y_p$ b) $y = y_c - y_p$
   c) linearly independent d) linearly dependent.

7. What is $L(t^n)$
   a) $\frac{n}{s^n}$ b) $\frac{s^{n+1}}{n!}$ c) $\frac{n!}{s^{n+1}}$ d) $\frac{1}{s^n}$

8. Find $L^{-1}\left(\frac{1}{s^3}\right) = ?$
   a) $\frac{t^2}{2}$ b) $\frac{t^3}{3}$ c) $\frac{t^4}{4}$ d) t

9. The convolution of two functions $(f * g)$ is defined by τ
   a) $(f * g)(t) = \int_0^\infty f(t)\, g(t)\, dt$ b) $(f * g)(t) = \int_0^t f(t)\, g(t - \tau)d\,\tau$
   c) $(f * g)(t) = \int_0^t f(t - \tau)\, g(t)d\,\tau$ d) $(f * g)(t) = \int_0^t f(t - \tau)g(t - \tau)d\,\tau$

10. What is $L^{-1}[-t\, f(t)]$
    a) $F'(s)$ b) $F(s)$ c) $F''(s)$ d) $F(0)$

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SECTION - B (35 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** Marks (5 × 7 = 35)

11. a) Solve the Initial value problem $\frac{dy}{dx} = 2x + 3, y(1) = 2$.

(OR)

b) Solve the Initial value problem $x^2 \frac{dy}{dx} + xy = \sin x, y(1) = y_0$.

12. a) State and prove principle of superposition for homogeneous equations?

(OR)

b) Show that the functions $y_1(x) = e^{-3x}, y_2(x) = \cos 2x$ and $y_3(x) = \sin 2x$ are linearly independent?

13. a) Show that the first three solutions $y_1(x) = x, y_2(x) = x \ln x$ and $y_3(x) = x^2$ of the third order equation $x^3 y^{(3)} - x^2 y'' + 2xy' - 2y = 0$ are linearly independent, To find the particular solution of given equation.

(OR)

b) Find the particular solution of $3y'' + y' - 2y = 2\cos x$.

14. a) Find $L[\cosh kt]$.

(OR)

b) Solve the Initial value problem $x'' + 4x = \sin 3t; x(0) = x'(0) = 0$.

15. a) Find $L^{-1}\left[\tan^{-1}\left(\frac{1}{s}\right)\right]$

(OR)

b) Find $L^{-1}\left[\frac{2s}{(s^2-1)^2}\right]$

SECTION -C (30 Marks)
Answer **ANY THREE** questions
ALL questions carry **EQUAL** Marks (3 × 10 = 30)

16. Find the general solution of $(x^2 + 1)\frac{dy}{dx} + 3xy = 6x$.

17. To solve the Initial value problem, $y'' + 2y' + y = 0; y(0) = 5, y'(0) = -3$.

18. Find a particular solution of the equation, $y'' + y = \tan x$.

19. Solve the Initial value problem $y'' + 4y' + 4y = t^2; y(0) = y'(0) = 0$.

20. Find $L[f(t)]$ if $f(t) = \begin{cases} \cos 2t & if\ 0 \le t < 2\pi \\ 0 & if\ t \ge 2\pi \end{cases}$

Z-Z-Z END