23MCU524B

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2025
(Fifth Semester)

Branch – **MATHEMATICS WITH COMPUTER APPLICATIONS**

MAJOR ELECTIVE COURSE- I: NUMBER THEORY

Time: Three Hours Maximum: 50 Marks

SECTION-A (5 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (5 x 1 = 5)

1. If the successors of two elements $a, b \in N$ Are equal then
   (i) $a < b$ (ii) $a > b$
   (iii) $a = b$ (iv) $a \neq b$

2. Positive integers greater than 1 which are not prime numbers are called
   (i) Composite numbers (ii)Integers
   (iii) Natural numbers (iv) Twin primes

3. Taking $(m-1)$ in the place of m the form $6m+5$ is equivalent of the form
   (i) $6m+1$ (ii) $6m-1$
   (iii) $6m$ (iv) $6m^2+2$

4 If p is a prime and $d/(p-1)$ then the number of least residues $mod\ p$ with order d is
   (i) $\Phi(d)$ (ii) $\Phi(p-1)$
   (iii) $\Phi(p)$ (iv) $\Phi(a)$

5. If the positive integers x, y, z satisfy the equation $x^3 + y^3 = z^3$ then at least one of the x, y, z is divisible by
   (i) 2 (ii) 3
   (iii) 4 (iv) 5

SECTION - B (15 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 3 = 15)

6 a. Prove by Mathematical induction that $3^{2n+1} + 2^{n+1} = M(7)$?

OR

b. State and prove Trichotomy law on natural numbers?

7 a. If n is a positive integer then prove that $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots+\frac{1}{n}$ is not an integer?

OR

b. Prove that $f_{n+1}^2 - f_n f_{n+2} = (-1)^n$ where f_n is the n^{th} Fibonacci number?

8 a. If $n > 1$ and $a^n - 1$ is prime then prove that $a = 2$ and n is a prime?

OR

b. Prove that the relation congruence modulo m is an equivalence in the set of integers?

Cont...

9 a. If n is prime then prove that nC_r is divisible by ?

OR

b. If a and b are relatively prime to 91 prove that $a^{12} - b^{12}$ is divisible by 91?

10 a. In a primitive solution of $x^2 + y^2 = z^2$ then prove that x and y are of different parity?

OR

b. If p is a prime of the form $4k + 3$ and p / n then prove that n has no proper representation?

SECTION -C (30 Marks)

Answer **ALL** questions

ALL questions carry **EQUAL** Marks (5 x 6 = 30)

11 a. Prove that any two integers a and b have a unique greatest common divisor?

OR

b. (i). Prove that $(a^m)^n = a^{mn}$; $a, m, n \in N$?

(ii). Prove that' $>$ 'is an order relation in N?

12 a. State and prove unique factorization theorem?

OR

b. Verify that $f_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$?

13 a.Prove that the Fermat numbers are Co-primes?

OR

b. If $(a, m) / b$ then show that $ax \equiv b(mod\, m)$ has exactly (a, m) solutions?

14 a. State and prove Fermat's theorem?

OR

b. State and prove Wilson's theorem?

15 a. Prove that the equations $x^4 + y^4 = z^4$ has no solution in positive integers?

OR

b. Prove that every prime p of the form $4k + 1$ is representable as a sum of two squares?

Z-Z-Z END